



Methodology

*Unified Interest Rate Model  
for European Securitisations*

OCTOBER 2015



*Insight beyond the rating.*

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# Unified Interest Rate Model for European Securitisations

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## Introduction

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This report describes the DBRS methodology for generation of consistent interest rate stresses across currency markets (CDN, CHF, DKK, EUR, GBP, JPY, NOK, SEK, and USD)<sup>1</sup> as well as the following markets which do not have underlying term structures (CP, PRIME, COFI).

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## Purpose

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The purpose of the DBRS Unified Interest Rate Model is to provide a consistent platform for interest rate stresses used in the rating process for structured finance transactions<sup>2</sup>. This system generates upwards and downwards stresses for such diverse categories as US LIBOR, EURIBOR, JPY, GBP, CP, PRIME, etc. Several of these categories feature interest rate curves directly observable on a daily basis and some are generated or set directly. Interest rate stresses for CP, COFI, PRIME, etc which are set, or reference rates, will be modeled in the Unified Interest Rate Model as a “best fit” rate using historical regressions of markets featuring daily interest rate curves.

Overall, the key feature of the model is the underlying framework which allows the swap curve of each currency to be processed and analysed uniformly.

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## Form of the Unified Interest Rate Model

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Much attention is focused on log-normal interest rate models simply because rates can never go negative due to the fact that the log of a negative number is undefined. On this basis alone, many log-normal models have been developed and utilised without considering whether or not the actual interest rates follow a log-normal process.

For the current version of the UIRM, DBRS has chosen to employ a “normal” model for reasons explained below. While negative rates are indeed possible in a normal model, DBRS establishes a tenor-based floor values to restrict the “negativeness” of the generated rates.

The advantage of using a normal model lies in the ease of calibrating a drift term such that various zero prices, zero rates, coupon paying bonds, and other instruments in the market can be priced correctly when analysing each instrument over the interest rate paths generated by the model.

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## Guide to the Generation of Interest Rate Stresses

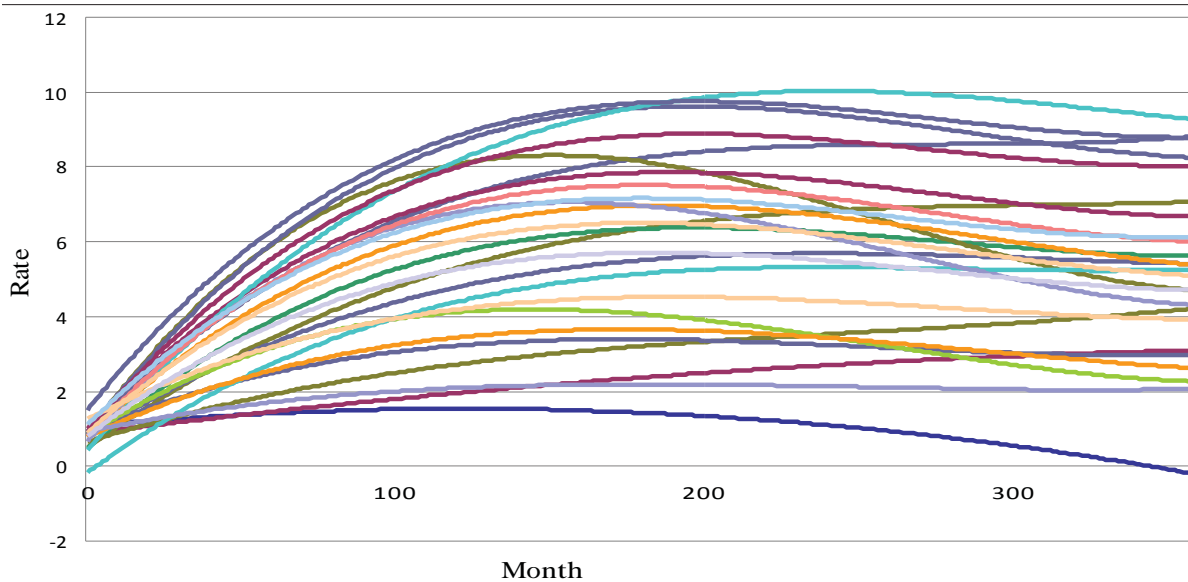
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The following are the steps involved in producing an interest rate stress matrix suitable for a prospective transaction having a USD LIBOR interest rate component. Obtaining the most current closing USD Swap curve, say from 06/29/2010, as well as the USD Cap/Floor Implied Volatilities from the Bloomberg VOLS page allows the process to begin.

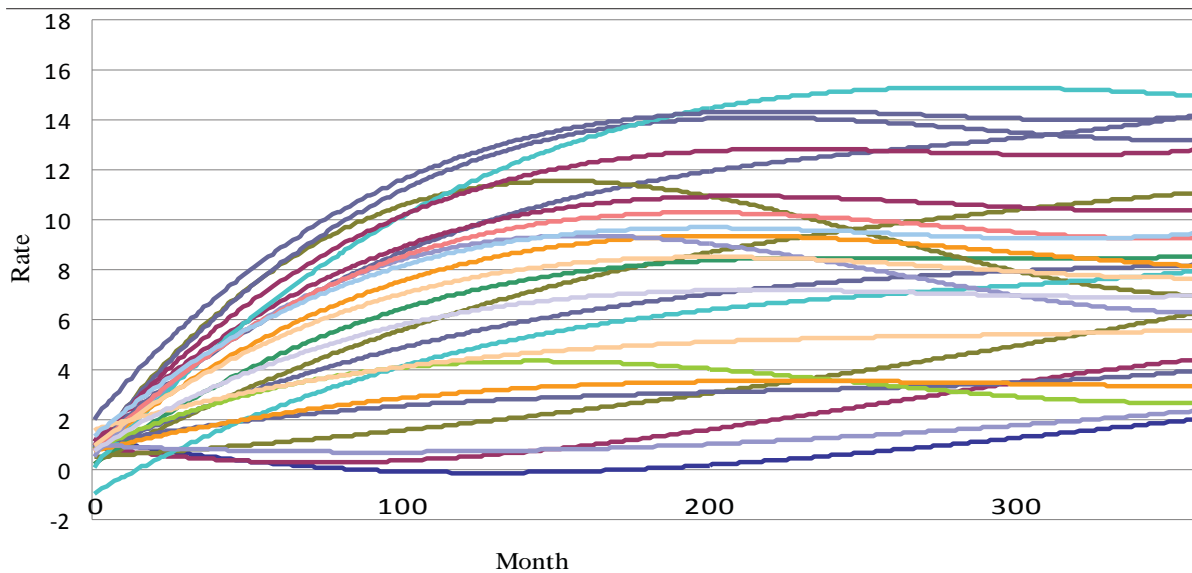
As this is a quick guide, we ignore the technical details and instead present the salient features.

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1. Canadian Dollar Swaps – CDN; Swiss Frank Swaps – CHF; Danish Krone Swaps- DKK; Euro Swaps – EUR; Pound Sterling Swaps – GBP; Japanese Yen Swaps – JPY; Norwegian Krone Swaps – NOK; Swedish Krone Swaps - SEK; US Dollar Swaps – USD.
  2. The methodology described applies to the securitisation of residential mortgages, auto loans, trade receivables, leases, secured and unsecured consumer loans, small to medium-sized enterprise loans and other corporate debt, or other structured finance transactions generally referred to as asset-backed securities . It may also be applied by DBRS to other types of structured finance transactions including commercial mortgage-backed securitisation transactions, arbitrage collateralised debt obligations, balance sheet collateralised loan obligations, and covered bonds.

Base Case Vol. - 1.0 Multiplier



Stressed Vol. - 1.75 Multiplier



Using interest rate cap volatilities from the Bloomberg VOLS page as well as the closing swap curve for the selected date, we are able to produce a specified number of interest rate paths mapping the course of 1 Month LIBOR, for example, over 360 months. Stressing the volatilities, which are extant for 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, and 20 year horizons, by multiplying by a 1.75 factor and re-running the paths, produces a bushier tree filling a wider span of rates. In the charts above, we can see that the 1.75 multiplier filled essentially the space from 0% to 15% while the 1.0 multiplier filled the rate space from 0% to 10%.

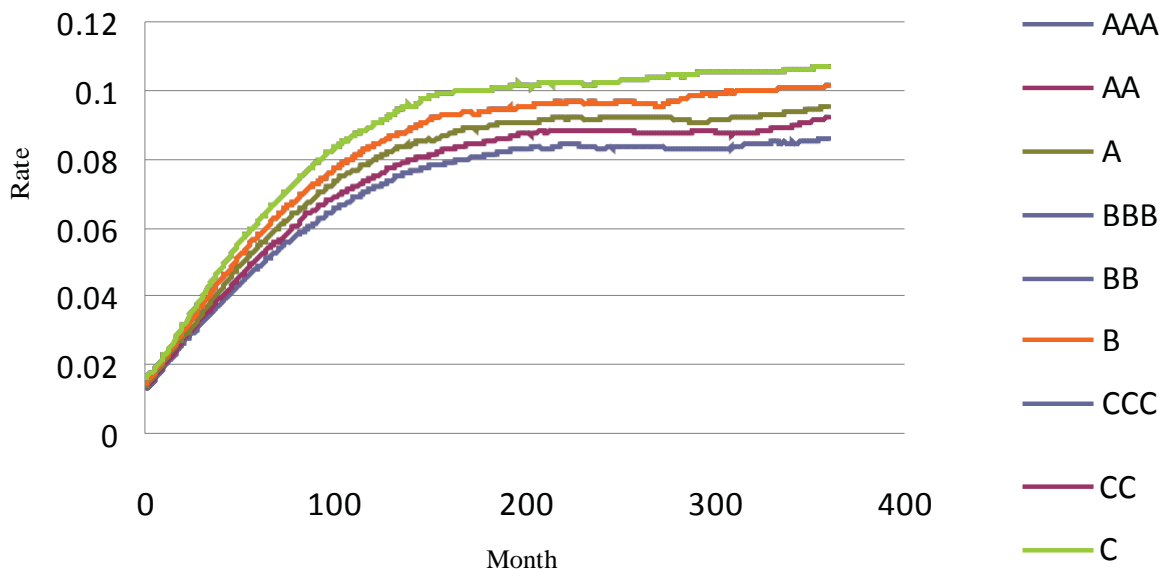
Once the interest rate paths are drawn for the issue under investigation, we imagine that a vertical line is drawn at each monthly node. Such a line must intersect with each interest rate path. Ignoring the negative rates below the established floor values (again see the discussion of Log-Normal vs Normal models), we rank the intersections for a given month, and select the value meeting the confidence level for the desired rating. More simply, imagine that



there were 1000 paths and 1000 intersections for month 36. If the AAA rating requires a 99.4% confidence interval, we would rank the 1000 values, calculate and select the 6th highest value.

No matter the specific path, we always rank all values over all paths each month and select the 6th highest value as the stressed AAA curve for consideration. Applying this same technique for the various confidence levels by rating produces the following chart for USD 1 Month LIBOR in the upward market direction:

Rating Curves - 1.75 Multiplier



Summarising, the 1 Month USD LIBOR stressed curves are produced from a closing market Swap curve, an observed volatility curve, and one stress multiplier. Following a monthly ranking, the stressed curves are manifest.

### EXTENSIBILITY

Having obtained the USD LIBOR swap curve and the related market volatility, it is possible to produce stressed interest rate curves for 1 month, 3 month, 6 month, 12 month, 2 year, 3 year, 5 year, 7 year, 10 year, and 30 year indices as well as values in between.

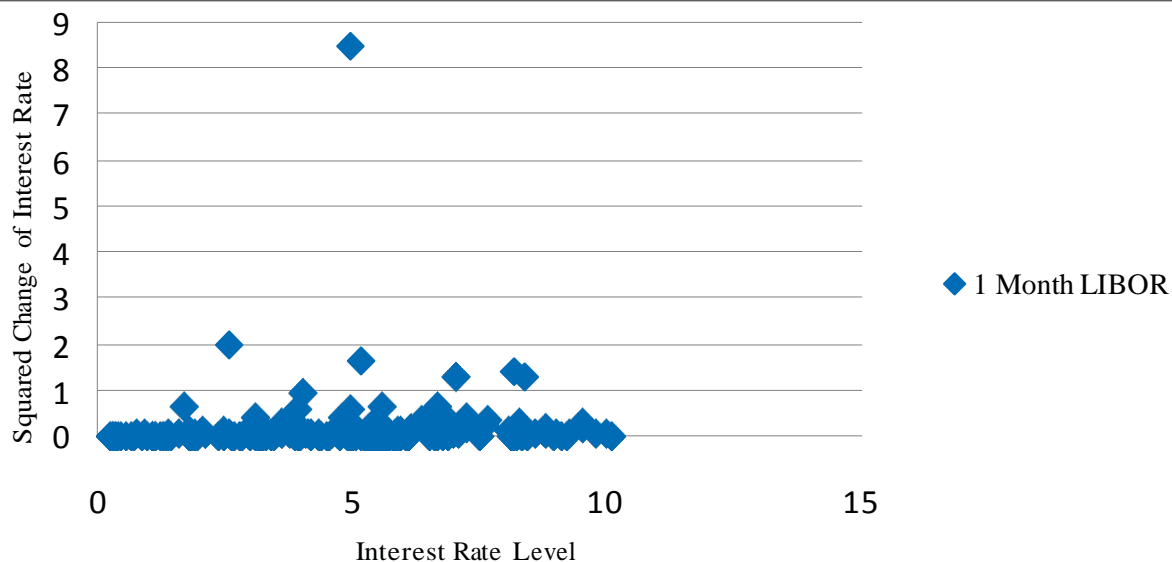
Obtaining similar swap curves and volatilities permits this approach to be readily extended to indices such as JPY LIBOR, GBP LIBOR, etc. Indices such as CP, PRIME, etc. may be handled via correlations to markets where swap curves and volatilities are extant.

## Technical Background

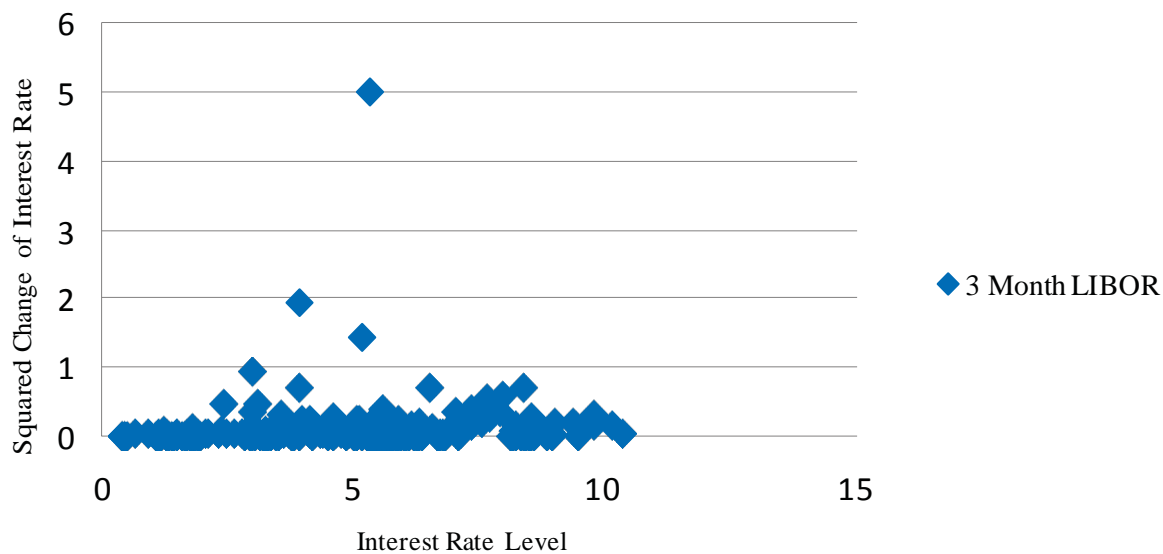
As previously stated, many times interest rate models of the Log-Normal variety are selected simply because they insure that negative rates are never encountered. If rates followed a log-normal pattern as well, the rationale for this type of model would be enhanced. The Oxford Guide to Financial Modeling: Applications for Capital Markets by Ho and Yi illustrates several scatter charts which display the squared change of interest rates vs. the interest rate level. Ho and Yi remark that the scatter plots do not indicate any specific relationship between interest rate levels and volatility and therefore conclude that they can not reject the normal interest rate models.

Using monthly data from 1987 to 2010, the scatter plots of Ho and Yi have been recalculated and are displayed below:

Volatility vs 1 Month LIBOR

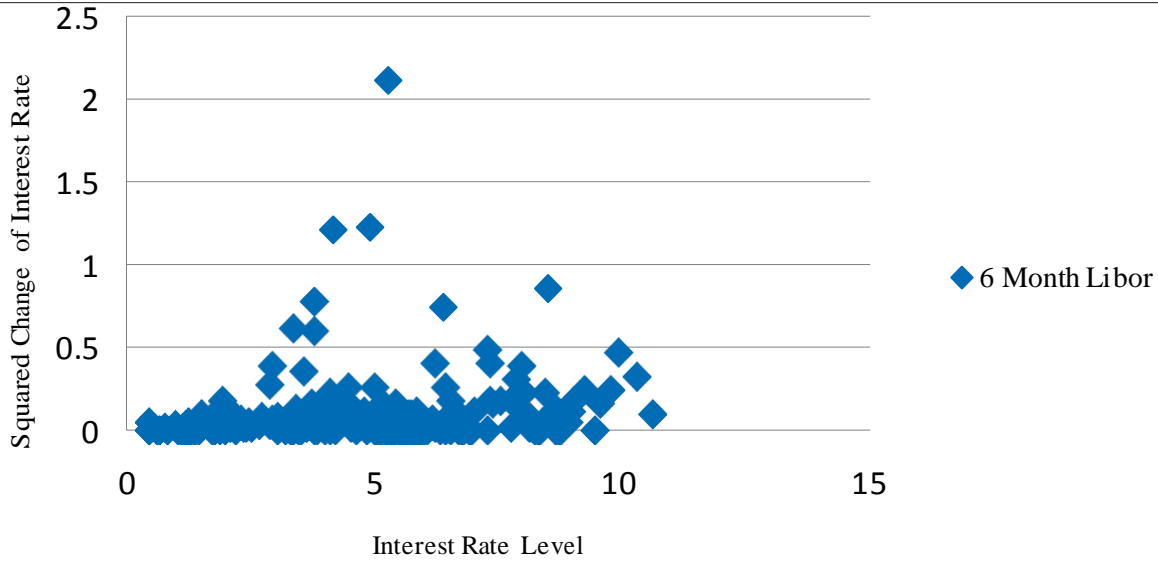


Volatility vs 3 Month LIBOR

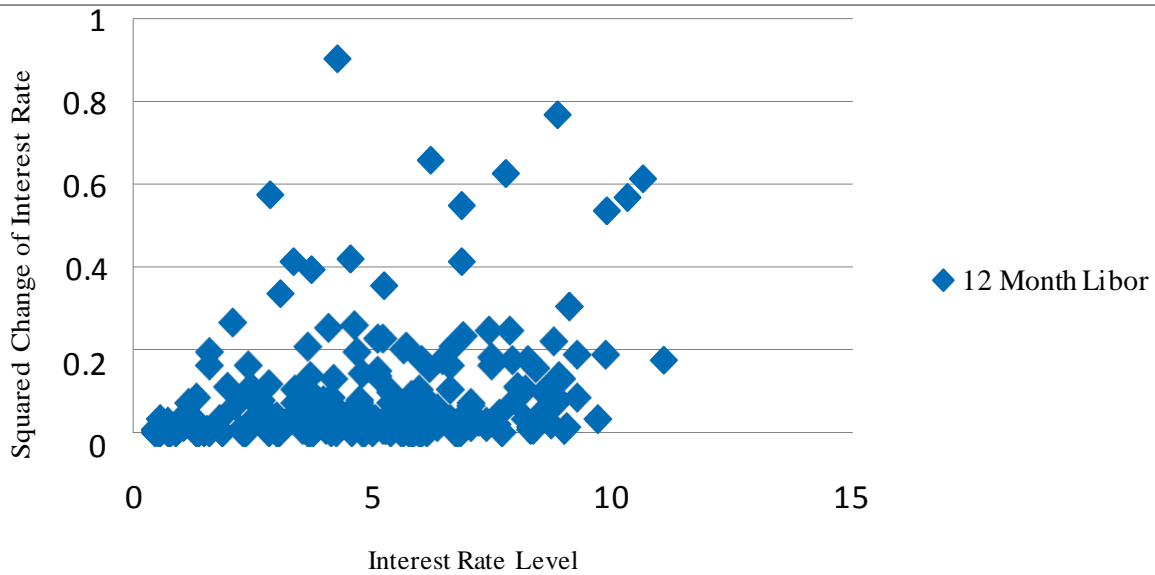




Volatility vs 6 Month LIBOR

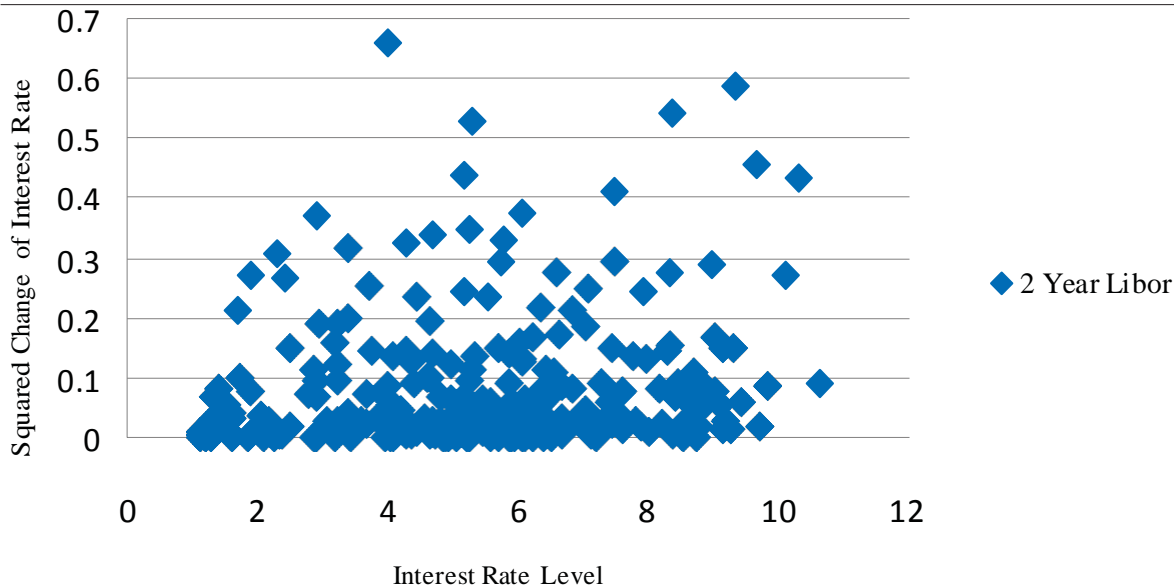


Volatility vs 12 Month LIBOR

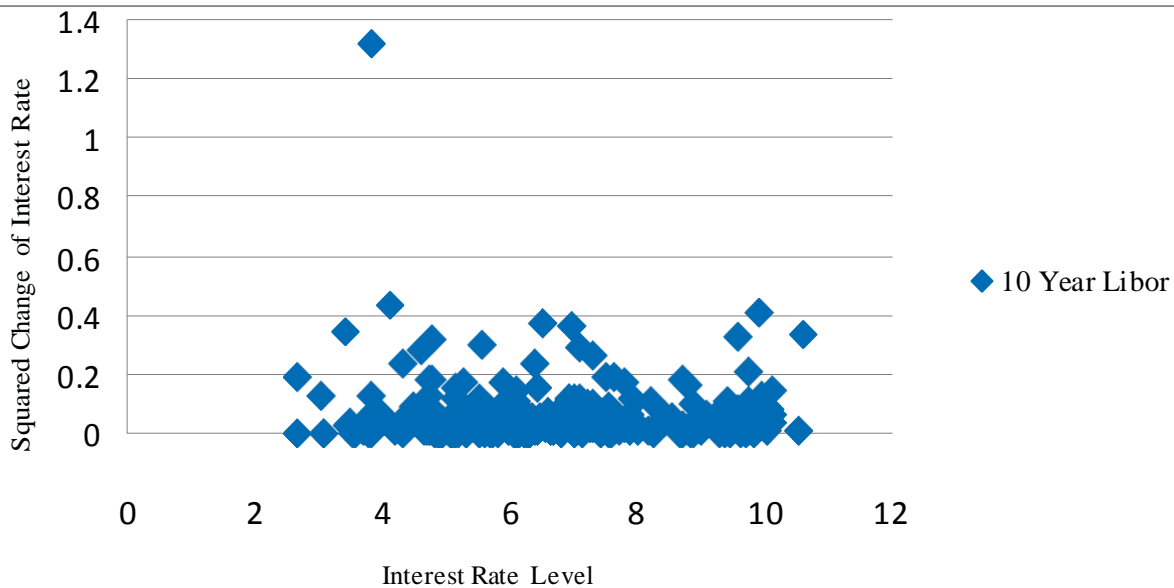




Volatility vs 2 Year LIBOR



Volatility vs 10 Year LIBOR



Clearly, none of the above plots suggest a positive correlation implying that in the time period from 1987 through the first half of 2010, the interest rate process for USD LIBOR has not been of the lognormal variety and therefore it can be concluded that volatility does not depend upon the level of interest rates. Historically periods of high interest rates were more likely to feature higher volatility and charts of such time periods would be more likely to display patterns indicating correlation between volatility and interest rates. Thus, the interest rate spectrum over recent history at least, can be evaluated using a normal model.

A no-arbitrage model of the short rate was proposed by Ho-Lee in 1986 in the following form

$$dr = \mu(r, t)dt + \sigma dZ \tag{1.1}$$



where variables have the following connotations:  $r$  is the level of interest rate,  $dr$  represents a small change in  $r$ ,  $\mu$  signifies a function defining the change in rate  $r$  for a particular time  $t$ ,  $dt$  signifies a small change in time,  $\sigma$  represents the volatility, and  $dZ$  indicates a standard Wiener process sometimes called Brownian motion.

The Ho-Lee model can be characterised as a “normal” model because the formulation does not depend upon the actual level of interest rates. Black-Karasinski 1991 modified the above process by recasting the equation as:

$$\frac{dr}{r} = \mu(r, t)dt + \sigma dZ \quad 1.2$$

Since  $\int \frac{dr}{r} = \ln(r)$  and  $\ln \in (0, \infty)$ , interest rates produced by this model can never be negative.

In equation 1.1, the function  $\mu(r, t)$ , which represents the change in rates with respect to time, can be

modified to include mean reversion providing a mechanism to bring rates back toward their central or average values.

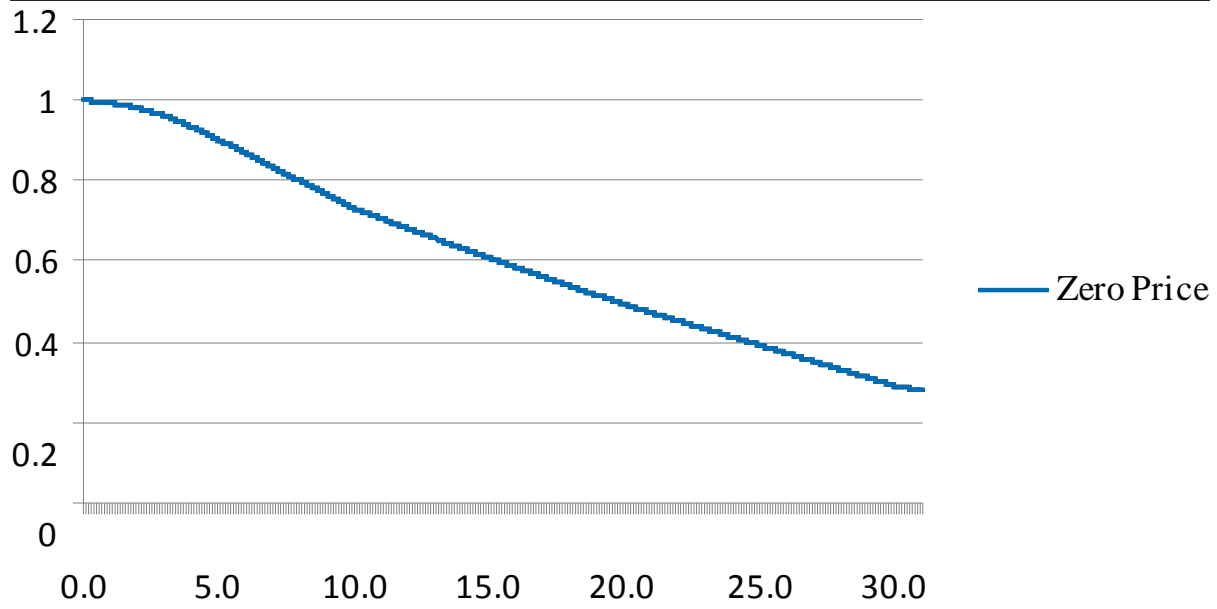
The main reason the normal model was utilised for the DBRS Unified Interest Rate Model stems from the notion that the bonds, and indeed cash flows, may be valued in terms of the interest rates and zero prices suggested by the model. This is not necessarily possible with a log-normal model.

## HIGHLIGHTS AND VALIDATION

Beginning with the premise that we wish to produce interest rate curves for USD LIBOR, the model must be presented with the swap curve on-the-run issues for the requested closing date as well as the interest rate cap volatilities.

Given that there exists an input value for 1 Year LIBOR and an input value for 2 Year LIBOR, the 1 Year rate, one year forward, must also be known otherwise there would be arbitrage along the curve. Applying this principal on a monthly and even on a daily level, allows the modeler to generate all possible rates of the form 1 day rate X days forward where X may be any number of days from 1 through 11,000 which is equivalent to the number of days in 30 years. The zero prices allow one to know the present value of \$1.00 on any future date.

Zero Price vs. Time in Years





One method to validate the bootstrapping process is to use the zero prices to compute the coupons on the reference input bond yields. Because the on-the-run issues are assumed to be priced at par, the yield and the coupon should be found to be identical upon review.

Due to reinvestment, determining the coupon on an issue with a maturity greater than 12 months differs from the more simplistic formula for calculation of the “rate” or “coupon for an investment with a time horizon less than 12 months. For such an investment, the beginning date, ending date, and corresponding zero prices are inserted in the following analysis:

$$rate = 36000 * (z[t_{beginning}] / z[t_{end}] - 1) / (t_{beginning} - t_{end}) \quad 1.3$$

where 36000 represents 360 days per year multiplied by 100 to change percent to a decimal basis.  $z$  denotes the zero price at some time  $t$  and  $(t_{beginning} - t_{end})$  refers to the number of days in the interval.

Addressing the first issue in the table below, to verify the 3 Month USD LIBOR rate using the calculated zero prices, we recall that the entered curve was for the close of business 6/29/2010 implying a settle date for the 3 month borrowing period of 7/02/2010. The conclusion of the 3 months borrowing period would fall on 10/02/2010 which happens to be a Saturday and therefore the borrowing period will be over 94 days from 7/02/2010 – 10/04/2010. The zero price corresponding to 7/02/2010 is 0.9999761116817766 and the zero price for 10/4/2010 is 0.9983598779682215. Plugging these values into formula (1.3) produces a rate of 0.619999999999799 compared to the input value of 0.6200 demonstrating a high degree of accuracy. Corresponding values can be seen to lead to a 6 month rate by calculation of 0.7500000000000092 vs. the input value of 0.7500 and to a 12 month rate by calculation of 0.7300000000000014 vs. the input value of 0.7300.

Validation of coupons applying to investments longer than 12 months requires the analysis of each semi-annual period separately. That said, each period is computed in a similar fashion to the short term investments but the time difference multiplied by the ending zero price is aggregated. Finally, the difference between the zero price relating to the settle date and the zero price for the investment maturity is divided by the aggregate value and is finally multiplied by 36000 to arrive at the coupon rate.

Using the 24 month or 2 year investment as an example, the settle date is 7/20/2010 placing the end of the first 6 month period on a Sunday which is then adjusted to 1/03/2011 corresponding to a zero price of 0.9961368342995802 which when multiplied by the 181 elapsed days produces the first of the values for aggregation of 180.3007670082240281. The second period begins on 1/03/2011 and ends 12 months from the settle date, again on Sunday, adjusted to Monday 7/04/2011, comprising 181 days, multiplied by the zero price for 7/04/2011 of 0.9925893171275642 equals 179.6586664000891176 which is added to our tally which now aggregates to 359.9594334083131457. The third period, 18 months from the settle date, ends on 1/02/2012, has a corresponding zero of 0.9865905271996760, and multiplying by the 178 days in the period, adds 175.6131138415423152 to our aggregate total. The final period ends 180 days later on 7/02/2012 and when its zero price of 0.9805917372717877 is multiplied by the 180 term of this period 176.5065127089218038 is added to the aggregation producing a total aggregated value of 712.0790599587772931.

In order to find the 2 year coupon, we compute the quotient in which the numerator is equal to 36000 times the difference in the zero price for the settle date and the zero price for the maturity date and the denominator is the aggregated value just calculated.

$$\begin{aligned} bcpn &= 36000 * (z[0] - z[mty]) / z_{tot} \\ &= 36000 * (0.9999761116817766 - 0.9805917372717877) / 712.0790599587772931 \\ &= 0.9799999999999960 \end{aligned}$$

compared to the input 2 year coupon of 0.9800.

Similarly, the computation process of bootstrapping the zero prices can be validated by calculating the implied coupon and comparing against the input coupon for the corresponding investment period.

**Table 1. Validation of On-The-Run Coupons**

VERIFICATION OF 3 MONTH USD LIBOR COUPON

begin	end	diff	beginning zero	ending zero	diff * end zero	summation
20100702	20101004	94.0000	0.9999761116817766	0.9983598779682215	93.8458285290128202	93.8458285290128202
$bcpn=36000*(z[0]/z[mty]-1.0)/diff=36000*(0.9999761116817766/0.9983598779682215-1.0)/94.0000000000000000=0.61999999999999799$						
003 maturity has a coupon of 0.620000						

VERIFICATION OF 6 MONTH USD LIBOR COUPON

begin	end	diff	beginning zero	ending zero	diff * end zero	summation
20100702	20110103	185.0000	0.9999761116817766	0.9961368342995802	184.2853143454223357	184.2853143454223357
$bcpn=36000*(z[0]/z[mty]-1.0)/diff=36000*(0.9999761116817766/0.9961368342995802-1.0)/185.0000000000000000=0.7500000000000092$						
006 maturity has a coupon of 0.750000						

VERIFICATION OF 12 MONTH USD LIBOR COUPON

begin	end	diff	beginning zero	ending zero	diff * end zero	summation
20100702	20110704	367.0000	0.9999761116817766	0.9925893171275642	364.2802793858160157	364.2802793858160157
$bcpn=36000*(z[0]/z[mty]-1.0)/diff=36000*(0.9999761116817766/0.9925893171275642-1.0)/367.0000000000000000=0.7300000000000014$						
012 maturity has a coupon of 0.730000						

VERIFICATION OF 24 MONTH USD LIBOR COUPON

begin	end	diff	beginning zero	ending zero	diff * end zero	summation	
20100702	20110103	181.0000	0.9999761116817766	0.9961368342995802	180.3007670082240281	180.3007670082240281	
20110103	20110704	181.0000	0.9961368342995802	0.9925893171275642	179.6586664000891176	359.9594334083131457	
20110704	20120102	178.0000	0.9925893171275642	0.9865905271996760	175.6131138415423152	535.5725472498554609	
20120102	20120702	180.0000	0.9865905271996760	0.9805917372717877	176.5065127089218038	712.0790599587772931	
		z[0]	z[mty]	z[0]-z[mty]	36000*(z[0]-z[mty])	ztot	36000*(z[0]-z[mty])/ztot
		0.9999761116817766	0.9805917372717877	0.0193843744099889	697.8374787595988664	712.0790599587772931	0.9799999999999960
$bcpn=36000*(z[0]-z[mty])/ztot=36000*(0.9999761116817766-0.9805917372717877)/712.0790599587772931=0.9799999999999960$							
024 maturity has a coupon of 0.980000							

VERIFICATION OF 36 MONTH USD LIBOR COUPON

begin	end	diff	beginning zero	ending zero	diff * end zero	summation	
20100702	20110103	181.0000	0.9999761116817766	0.9961368342995802	180.3007670082240281	180.3007670082240281	
20110103	20110704	181.0000	0.9961368342995802	0.9925893171275642	179.6586664000891176	359.9594334083131457	
20110704	20120102	178.0000	0.9925893171275642	0.9865905271996760	175.6131138415423152	535.5725472498554609	
20120102	20120702	180.0000	0.9865905271996760	0.9805917372717877	176.5065127089218038	712.0790599587772931	
20120702	20130102	180.0000	0.9805917372717877	0.9706269306863092	174.7128475235356575	886.7919074823129222	
20130102	20130702	180.0000	0.9706269306863092	0.9608245937734201	172.9484268792156172	1059.7403343615285394	
		z[0]	z[mty]	z[0]-z[mty]	36000*(z[0]-z[mty])	ztot	36000*(z[0]-z[mty])/ztot
		0.9999761116817766	0.9608245937734201	0.0391515179083565	1409.4546447008328869	1059.7403343615285394	1.3299999999999998
$bcpn=36000*(z[0]-z[mty])/ztot=36000*(0.9999761116817766-0.9608245937734201)/1059.7403343615285394=1.3299999999999998$							
036 maturity has a coupon of 1.330000							

VERIFICATION OF 60 MONTH USD LIBOR COUPON

begin	end	diff	beginning zero	ending zero	diff * end zero	summation
20100702	20110103	181.0000	0.9999761116817766	0.9961368342995802	180.3007670082240281	180.3007670082240281
20110103	20110704	181.0000	0.9961368342995802	0.9925893171275642	179.6586664000891176	359.9594334083131457
20110704	20120102	178.0000	0.9925893171275642	0.9865905271996760	175.6131138415423152	535.5725472498554609
20120102	20120702	180.0000	0.9865905271996760	0.9805917372717877	176.5065127089218038	712.0790599587772931
20120702	20130102	180.0000	0.9805917372717877	0.9706269306863092	174.7128475235356575	886.7919074823129222
20130102	20130702	180.0000	0.9706269306863092	0.9608245937734201	172.9484268792156172	1059.7403343615285394
20130702	20140102	180.0000	0.9608245937734201	0.9472730654349665	170.5091517782939547	1230.2494861398224657
20140102	20140702	180.0000	0.9472730654349665	0.9339424859281182	168.1096474670612793	1398.3591336068836881
20140702	20150102	180.0000	0.9339424859281182	0.9174500567089593	165.1410102076126805	1563.5001438144963686



20150102 20150702 180.0000 0.9174500567089593 0.9012265257922865 162.2207746426115591 1725.7209184571079277  
z[0] z[mtly] z[0]-z[mtly] 36000\*(z[0]-z[mtly]) ztot 36000\*(z[0]-z[mtly])/ztot  
0.9999761116817766 0.9012265257922865 0.0987495858894901 3554.9850920216445047 1725.7209184571079277 2.0600000000000014  
bcpn=36000\*(z[0]-z[mtly])/ztot=36000\*(0.9999761116817766-0.9012265257922865)/1725.7209184571079277=2.0600000000000014  
060 maturity has a coupon of 2.060000

VERIFICATION OF 10 YEAR USD LIBOR COUPON

begin	end	diff	beginning zero	ending zero	diff * end zero	summation
20100702	20110103	181.0000	0.9999761116817766	0.9961368342995802	180.3007670082240281	180.3007670082240281
20110103	20110704	181.0000	0.9961368342995802	0.9925893171275642	179.6586664000891176	359.9594334083131457
20110704	20120102	178.0000	0.9925893171275642	0.9865905271996760	175.6131138415423152	535.5725472498554602
0120102	20120702	180.0000	0.9865905271996760	0.9805917372717877	176.5065127089218038	712.0790599587772931
20120702	20130102	180.0000	0.9805917372717877	0.9706269306863092	174.7128475235356575	886.7919074823129222
20130102	20130702	180.0000	0.9706269306863092	0.9608245937734201	172.9484268792156172	1059.7403343615285394
20130702	20140102	180.0000	0.9608245937734201	0.9472730654349665	170.5091517782939547	1230.2494861398224657
20140102	20140702	180.0000	0.9472730654349665	0.9339424859281182	168.1096474670612793	1398.3591336068836881
20140702	20150102	180.0000	0.9339424859281182	0.9174500567089593	165.1410102076126805	1563.5001438144963686
20150102	20150702	180.0000	0.9174500567089593	0.9012265257922865	162.2207746426115591	1725.7209184571079277
20150702	20160104	182.0000	0.9012265257922865	0.8845920209676383	160.9957478161101676	1886.7166662732181521
20160104	20160704	180.0000	0.8845920209676383	0.8683152474295418	156.2967445373175224	2043.0134108105357882
20160704	20170102	178.0000	0.8683152474295418	0.8501196320961744	151.3212945131190281	2194.3347053236548163
20170102	20170703	181.0000	0.8501196320961744	0.8319240167628068	150.5782470340680277	2344.9129523577230430
20170703	20180102	179.0000	0.8319240167628068	0.8158323092703335	146.0339833593896799	2490.9469357171128649
20180102	20180702	180.0000	0.8158323092703335	0.7999164674335155	143.9849641380328080	2634.9318998551457298
20180702	20190102	180.0000	0.7999164674335155	0.7829861083393025	140.9374995010744556	2775.8693993562201285
20190102	20190702	180.0000	0.7829861083393025	0.7663317877085819	137.9397217875447552	2913.8091211437649690
20190702	20200102	180.0000	0.7663317877085819	0.7487815246253127	134.7806744325562818	3048.5897955763211939
20200102	20200702	180.0000	0.7487815246253127	0.7314220252712094	131.6559645488176784	3180.2457601251389860

z[0] z[mtly] z[0]-z[mtly] 36000\*(z[0]-z[mtly]) ztot 36000\*(z[0]-z[mtly])/ztot  
0.9999761116817766 0.7314220252712094 0.2685540864105672 9667.9471107804201893 3180.2457601251389860 3.0399999999999991  
bcpn=36000\*(z[0]-z[mtly])/ztot=36000\*(0.9999761116817766-0.7314220252712094)/3180.2457601251389860=3.0399999999999991  
120 maturity has a coupon of 3.040000

VERIFICATION OF 30 YEAR USD LIBOR COUPON

begin	end	diff	beginning zero	ending zero	diff * end zero	summation
20100702	20110103	181.0000	0.9999761116817766	0.9961368342995802	180.3007670082240281	180.3007670082240281
20110103	20110704	181.0000	0.9961368342995802	0.9925893171275642	179.6586664000891176	359.9594334083131457
20110704	20120102	178.0000	0.9925893171275642	0.9865905271996760	175.6131138415423152	535.5725472498554609
20120102	20120702	180.0000	0.9865905271996760	0.9805917372717877	176.5065127089218038	712.0790599587772931
20120702	20130102	180.0000	0.9805917372717877	0.9706269306863092	174.7128475235356575	886.7919074823129222
20130102	20130702	180.0000	0.9706269306863092	0.9608245937734201	172.9484268792156172	1059.7403343615285394
20130702	20140102	180.0000	0.9608245937734201	0.9472730654349665	170.5091517782939547	1230.2494861398224657
20140102	20140702	180.0000	0.9472730654349665	0.9339424859281182	168.1096474670612793	1398.3591336068836881
20140702	20150102	180.0000	0.9339424859281182	0.9174500567089593	165.1410102076126805	1563.5001438144963686
20150102	20150702	180.0000	0.9174500567089593	0.9012265257922865	162.2207746426115591	1725.7209184571079277
20150702	20160104	182.0000	0.9012265257922865	0.8845920209676383	160.9957478161101676	1886.7166662732181521
20160104	20160704	180.0000	0.8845920209676383	0.8683152474295418	156.2967445373175224	2043.0134108105357882
20160704	20170102	178.0000	0.8683152474295418	0.8501196320961744	151.3212945131190281	2194.3347053236548163
20170102	20170703	181.0000	0.8501196320961744	0.8319240167628068	150.5782470340680277	2344.9129523577230430
20170703	20180102	179.0000	0.8319240167628068	0.8158323092703335	146.0339833593896799	2490.9469357171128649
20180102	20180702	180.0000	0.8158323092703335	0.7999164674335155	143.9849641380328080	2634.9318998551457298
20180702	20190102	180.0000	0.7999164674335155	0.7829861083393025	140.9374995010744556	2775.8693993562201285
20190102	20190702	180.0000	0.7829861083393025	0.7663317877085819	137.9397217875447552	2913.8091211437649690
20190702	20200102	180.0000	0.7663317877085819	0.7487815246253127	134.7806744325562818	3048.5897955763211939



20200102	20200702	180.0000	0.7487815246253127	0.7314220252712094	131.6559645488176784	3180.2457601251389860	
20200702	20210104	182.0000	0.7314220252712094	0.7186530942863306	130.7948631601121861	3311.0406232852510584	
20210104	20210702	178.0000	0.7186530942863306	0.7063647144675496	125.7329191752238273	3436.7735424604748005	
20210702	20220103	181.0000	0.7063647144675496	0.6936644336492561	125.5532624905153511	3562.3268049509902085	
20220103	20220704	181.0000	0.6936644336492561	0.6811701033307189	123.2917887028601314	3685.6185936538504393	
20220704	20230102	178.0000	0.6811701033307189	0.6690211929576704	119.0857723464653333	3804.7043660003159857	
20230102	20230703	181.0000	0.6690211929576704	0.6568722825846218	118.8938831478165525	3923.5982491481327088	
20230703	20240102	179.0000	0.6568722825846218	0.6446566199567764	115.3935349722629695	4038.9917841203955504	
20240102	20240702	180.0000	0.6446566199567764	0.6325077095837278	113.8513877250710067	4152.8431718454667134	
20240702	20250102	180.0000	0.6325077095837278	0.6202252947010853	111.6405530461953646	4264.4837248916619501	
20250102	20250702	180.0000	0.6202252947010853	0.6081431365828338	109.4657645849100902	4373.9494894765721256	
20250702	20260102	180.0000	0.6081431365828338	0.5965205101194861	107.3736918215075065	4481.3231812980793620	
20260102	20260702	180.0000	0.5965205101194861	0.5850873830006496	105.3157289401169123	4586.6389102381963312	
20260702	20270104	182.0000	0.5850873830006496	0.5733384236409611	104.3475931026549262	4690.9865033408514137	
20270104	20270702	178.0000	0.5733384236409611	0.5620316294184653	100.0416300364868221	4791.0281333773382357	
20270702	20280103	181.0000	0.5620316294184653	0.5503458365069471	99.6125964077574366	4890.6407297850955729	
20280103	20280703	180.0000	0.5503458365069471	0.5388495429399403	96.9929177291892444	4987.6336475142852578	
20280703	20290102	179.0000	0.5388495429399403	0.5272900829247629	94.3849248435325592	5082.0185723578179022	
20290102	20290702	180.0000	0.5272900829247629	0.5158569558059263	92.8542520450667297	5174.8728244028843619	
20290702	20300102	180.0000	0.5158569558059263	0.5042343293425786	90.7621792816641459	5265.6350036845487921	
20300102	20300702	180.0000	0.5042343293425786	0.4928012022237420	88.7042164002735518	5354.3392200848220455	
20300702	20310102	180.0000	0.4928012022237420	0.4826596784315279	86.8787421176750172	5441.2179622024968921	
20310102	20310702	180.0000	0.4826596784315279	0.4726835055707087	85.0830310027275516	5526.3009932052245858	
20310702	20320102	180.0000	0.4726835055707087	0.4625419817784946	83.2575567201290312	5609.5585499253538728	
20320102	20320702	180.0000	0.4625419817784946	0.4525106919405437	81.4519245492978712	5691.0104744746513461	
20320702	20330103	181.0000	0.4525106919405437	0.4423140511711980	80.0588432619868513	5771.0693177366383679	
20330103	20330704	181.0000	0.4423140511711980	0.4322827613332472	78.2431798013177371	5849.3124975379560055	
20330704	20340102	178.0000	0.4322827613332472	0.4222514714952963	75.1607619261627349	5924.4732594641191099	
20340102	20340703	181.0000	0.4222514714952963	0.4122201816573454	74.6118528799795229	5999.0851123440988886	
20340703	20350102	179.0000	0.4122201816573454	0.4021337748422630	71.9819456967650666	6071.0670580408641399	
20350102	20350702	180.0000	0.4021337748422630	0.3921576019814437	70.5883683566598563	6141.6554263975240247	
20350702	20360102	180.0000	0.3921576019814437	0.3820160781892296	68.7628940740613217	6210.4183204715855027	
20360102	20360702	180.0000	0.3820160781892296	0.3719847883512788	66.9572619032301759	6277.3755823748160765	
20360702	20370102	180.0000	0.3719847883512788	0.3618432645590647	65.1317876206316413	6342.5073699954473341	
20370102	20370702	180.0000	0.3618432645590647	0.3518670916982454	63.3360765056841686	6405.8434465011314387	
20370702	20380104	182.0000	0.3518670916982454	0.3416153339517681	62.1739907792217963	6468.0174372803530787	
20380104	20380702	178.0000	0.3416153339517681	0.3317493950452121	59.0513923180477462	6527.0688295984009528	
20380702	20390103	181.0000	0.3317493950452121	0.3215527542758664	58.2010485239318172	6585.2698781223325568	
20390103	20390704	181.0000	0.3215527542758664	0.3115214644379155	56.3853850632627100	6641.6552631855956861	
20390704	20400102	178.0000	0.3115214644379155	0.3014901745999646	53.6652510787937018	6695.3205142643892032	
20400102	20400702	180.0000	0.3014901745999646	0.2914588847620138	52.4625992571624735	6747.7831135215519680	
	$z[0]$		$z[mty]$	$z[0]-z[mty]$	$36000*(z[0]-z[mty])$	$z_{tot}$	$36000*(z[0]-z[mty])/z_{tot}$
	0.9999761116817766		0.2914588847620138	0.7085172269197628	25506.6201691114620189	6747.7831135215519680	3.7799999999999994
	$bcpn=36000*(z[0]-z[mty])/z_{tot}=36000*(0.9999761116817766-0.2914588847620138)/6747.7831135215519680=3.7799999999999994$						
	360 maturity has a coupon of 3.780000						





The previous exposition demonstrated excellent alignment of zero prices and coupon determination for on-the-run issues. Using this zero curve, a specified number of paths will be generated and calibrated such that the entire path system will still price on-the-run issues correctly and allow a volatility grid to dictate the dispersion or “fan width” of the resulting interest rates.

In order to begin the path generation process, it is helpful to view the structure which allows calculation of zero prices, and hence all possible derivative information, for a given path including the generation of zero prices just detailed above. Referring to Table 2, a grid is presented covering 40 years. For each period or tick, a normal random number is detailed in column A followed by the corresponding volatility measure in column B. Column C, labeled “y-instantaneous” is the product of the random number and the square root of the volatility multiplied by 365 days per year. Column D is a running sum of Column C. A calibration factor, labeled stretch representing a drift calibration term is presented in Column E. Column F is the sum of an initial rate level, and Columns D and E. Finally, Column G is Column F multiplied by 365.

Using the grid, it is possible to know the zero price and zero rate for any tick point via the following steps:

$$\begin{aligned}\Delta t &= date_n - date_{n-1} \\ r_{n-1} &= yqume_{n-1} + stretch_{n-1} + delrc_{n-1} + r_0 \\ r_n &= yqume_n + stretch_n + delrc_n + r_0 \\ Z_n &= \exp(-(r_{n-1} + r_n) * \Delta t / 2) * Z_{n-1}\end{aligned}$$

Selection of different random numbers creates different grids which form the basis for all derived values on the different interest rate paths.

Without calibration, the matrix of generated paths will not price the on-the-run issues correctly. Therefore, on a tick by tick basis, beginning with the shortest duration, the grid structure is evaluated to determine the extent of the error and a calibrating factor is calculated and retained in order to eliminate pricing error.

As has been previous demonstrated, all calculations stem from the zero prices. Therefore having a calibrated grid of zero prices allows for accurate calculation of any investment of \$1 in the future as well as more complicated securities such as on-the-run and off-the-run issues.

Lacking from this amalgam of zeros and rates is the ability to price volatility dependent issues. To accurately price such issues it is critical to iterate on the volatility surface while calibrating the rate space, as we have done, to produce a series of independent paths that will correctly price volatility independent and volatility dependent securities.



**Table 2. Sample Grid Structure**

per date	A normal ()	B findvol output	C yinst	D yqume	E stretch	F y+r0+stretch	G yearly
0	0.000000000000	0.000000000000	0.000000000000	0.000000000000	0.000000000000	0.000011944302	0.436265622379
1 20100702	-0.172279924073	0.000000000003	-0.000099828387	-0.000099828387	0.000201063488	0.000012221658	0.004460905249
2 20100802	-0.836853802593	0.000000000039	-0.001909127026	-0.002008955413	0.001920904787	0.000011703067	0.004271619523
3 20100902	1.615439849006	0.000000000039	0.003685326952	0.001676371540	0.003584272380	0.000026357025	0.009620314068
4 20101004	0.187117448254	0.000000000040	0.000433704232	0.002110075772	0.004573701396	0.000030256020	0.011043447317
5 20110103	0.653144674658	0.000000000115	0.002552903926	0.004662979698	0.002790489325	0.000032364765	0.011813139171
6 20110704	-0.176774038977	0.000000000229	-0.000977143987	0.003685835711	0.007792004180	0.000043390438	0.015837510039
7 20120702	0.194145772837	0.000000000459	0.001517623751	0.005203459463	0.015921295669	0.000069820343	0.025484425280
8 20130702	-0.546363796255	0.000000000445	-0.004207231249	0.000996228214	0.023884561999	0.000080110850	0.029240460361
9 20140702	1.204320789901	0.000000000336	0.008060846947	0.009057075161	0.031117969711	0.000122012918	0.044534715021
10 20150702	0.925709109950	0.000000000236	0.005194122706	0.014251197867	0.032517571226	0.000140077916	0.051128439241
11 20160704	-1.355559859037	0.000000000175	-0.006547126262	0.007704071604	0.038554018331	0.000138678795	0.050617760083
12 20170703	1.530554839214	0.000000000139	0.006584897552	0.014288969157	0.035354940322	0.000147955013	0.054003579626
13 20180702	1.020178343547	0.000000000118	0.004042880621	0.018331849777	0.039021475827	0.000169076701	0.061712995752
14 20190702	0.051488940086	0.000000000104	0.000191874097	0.018523723874	0.042741193472	0.000179793390	0.065624587494
15 20200702	0.708497289638	0.000000000095	0.002514096798	0.021037820672	0.031857576849	0.000156863199	0.057255067670
16 20220704	-1.226160576778	0.000000000175	-0.005919639905	0.015118180767	0.033975846018	0.000146448485	0.053453696933
17 20250702	-0.789721313248	0.000000000228	-0.004348819632	0.010769361135	0.037096742069	0.000143084311	0.052225773353
18 20300702	0.871673288691	0.000000000326	0.005741488139	0.016510849274	0.041797156987	0.000171692264	0.062667676410
19 20400702	0.205602548047	0.000000000550	0.001760089243	0.018270938518	0.046160304009	0.000188468254	0.068790912675
20 20500630	0.332079300185	0.000000000550	0.002841643533	0.021112582051	0.051018112188	0.000209562642	0.076490364387



**Table 3. Calibration of tick 11 corresponding to 20160704**

[11] analytic zero for 20160704.0000 is 0.8683152474295418

path	zero[n-3]	zero[n-2]	zero[n-1]	zero[n]	avg(zero[n]*1.0)
0	0.9515548567786449	0.9208288469405340	0.8786947176960781	0.8355048353103056	0.8355048353103056
1	1.0057528135236631	1.0018744294209057	0.9888767403023380	0.9696378833872780	0.9025713593487918
2	0.9706146681010209	0.9509542924593867	0.9265495114137275	0.9062045760252507	0.9037824315742782
3	0.9106364064078173	0.8601377738650912	0.7990865686279695	0.7398144523978433	0.8627904367801694
4	0.9589672956736875	0.9459462355758765	0.9267187946542810	0.9024539520869809	0.8707231398415317
5	1.0151065949747466	1.0204976686970189	1.0310782991404288	1.0457176351458843	0.8998888890589237
6	0.9760302729143443	0.9478032947345858	0.9110691236859308	0.8712553887140699	0.8957983890096590
7	0.9092536738691235	0.8602121142589435	0.8058816593156289	0.7528995218139460	0.8779360306101949
8	0.9480664471021195	0.9061296139675257	0.8554340691600042	0.8090427625413040	0.8702812230469847
9	0.95458248783017146	0.9187945314413819	0.878069668556564	0.8370693000353222	0.8669600307458184
10	0.9541477397039242	0.9232499659546038	0.8860565168686584	0.8494878536133985	0.8653716510065075
11	1.0048719857232982	0.9926712272977087	0.9713046820776302	0.9486467312644391	0.8723112410280018
12	0.9410312072244159	0.8898155469979760	0.8229596673902773	0.7524260035284646	0.8630892996818835
13	0.9401032735956393	0.8965430924699366	0.8466524587482746	0.7960717580714554	0.8583023324239959
14	0.9537033943467573	0.9218289166613053	0.8849290281382252	0.8450248310556351	0.8574171656661052
15	0.9919817069802634	0.9855961920046116	0.9770091702534148	0.9645624925527243	0.8641137485965189
16	0.9569370597091332	0.9290164519894101	0.8980702082532771	0.8664070672853672	0.8642486496958629
17	0.9786248789819079	0.9602186957706069	0.9360197592977039	0.9064951623391474	0.8665956781760454
18	0.9223114446367479	0.8770570618867215	0.8307616468330447	0.7836242552469105	0.8622287611797752
19	0.9416425925221282	0.9134548252059055	0.8805285436705006	0.8501680108284214	0.8616257236622076
20	0.9673177998488416	0.9513454263159293	0.9375747024636375	0.9228276739245300	0.8645401022461277
480	0.9544918908560924	0.9178789120231019	0.8750499712806408	0.8322485340078112	0.8668400405408758
481	0.9757403189771565	0.9622073908997618	0.9367440524018146	0.9042867376475542	0.8669177307838357
482	0.9308477331795901	0.8869413969329909	0.8383858721374957	0.7909060182573451	0.8667603566378181
483	0.9532233441846450	0.9201888881060101	0.8766106876123528	0.8298504057924402	0.8666840964087988
484	0.9151134776478388	0.8554638955361670	0.7919364527822618	0.7286526957581743	0.8663994955827150
485	0.9795662955911203	0.9516189937461134	0.9133025925847316	0.8726211620205500	0.8664122973655090
486	0.9369475988482806	0.9113635649832024	0.8910566145264066	0.8803744343291627	0.8664409670512659
487	0.9524602115863845	0.9189784554695491	0.8718012203790302	0.8208322163049924	0.8663475064964580
488	0.9567360195017350	0.9235836456009039	0.8818070420846186	0.8409471008716937	0.8662955629266732
489	1.0108681038998930	1.0024453829993905	0.9844003374828131	0.9597741671481819	0.8664863355883498
490	0.9618672640936562	0.9386030289418842	0.9085696833842275	0.8751302374508861	0.8665039402764608
491	0.9336862246598268	0.8952050944364063	0.8532013498915735	0.8152857877443856	0.8663998383404199
492	0.9399421387676299	0.9063243332558951	0.8728164223197123	0.8378450058610281	0.8663419177877234
493	0.9415631164085818	0.9166257187761041	0.88856197711295429	0.8582305421625166	0.8663254979990084
494	0.9778211571974894	0.9638006877910213	0.9456917584422360	0.9238885882137970	0.8664417870701494
495	0.9697188117602353	0.9468225505465561	0.9216009751055934	0.8928725478446535	0.8664950748942916
496	0.9502043714269706	0.9165389846175833	0.8782589413646807	0.8415491946015173	0.8664448819761974
497	0.9598649338986657	0.9300169057199971	0.8973445770933661	0.8663393069671564	0.8664446699781874
498	0.9659910400882985	0.9442615517524653	0.9154982783962132	0.8831630852499108	0.8664781738164072
499	0.9607313858987854	0.9403811282971752	0.9136635495578103	0.8813054275211012	0.8665078283238167
500	0.9560403841219246	0.9247786269881655	0.8860238876961504	0.8452077925284386	0.8664653132823088

avg zero : 0.8664653132823088      analytical zero: 0.8683152474295418      diff: -0.0018499341472330      days: 368.0000      delrc: -0.0000115910841628

avgpx : 0.8664653132823088      ln: -0.1433332014617663      ln/4: -0.0358333003654416      analytical/4: -0.0353001104939511      diff: -0.0005331898714905



**Table 4. Verification of Calibration**

Recalculation using corrected delrc					
path	zero[n-3]	zero[n-2]	zero[n-1]	zero[n]	avg(zero[n]*1.0)
0	0.9515548567786449	0.9208288469405340	0.8786947176960781	0.8372886677399773	0.8372886677399773
1	1.0057528135236631	1.0018744294209057	0.9888767403023380	0.9717080946276261	0.9044983811838017
2	0.9706146681010209	0.9509542924593867	0.9265495114137275	0.9081393549066087	0.9057120390914041
3	0.9106364064078173	0.8601377738650912	0.7990865686279695	0.7413939824691888	0.8646325249358502
4	0.9589672956736875	0.9459462355758765	0.9267187946542810	0.9043807232533265	0.8725821645993455
5	1.0151065949747466	1.0204976686970189	1.0310782991404288	1.0479502793521385	0.9018101837248110
6	0.9760302729143443	0.9478032947345858	0.9110691236859308	0.8731155498420869	0.8977109503129933
7	0.9092536738691235	0.8602121142589435	0.8058816593156289	0.7545069889721671	0.8798104551453900
8	0.9480664471021195	0.9061296139675257	0.8554340691600042	0.8107700975079251	0.8721393042967828
9	0.9545837883017146	0.9187945314413819	0.8780696688556564	0.8388564726526192	0.8688110211323664
10	0.9541477397039242	0.9232499659546038	0.8860565168686584	0.8513015402826385	0.8672192501460274
11	1.0048719857232982	0.9926712272977087	0.9713046820776302	0.9506721256511792	0.8741736564381234
12	0.9410312072244159	0.8898155469979760	0.8229596673902773	0.7540324597083669	0.8649320259204498
13	0.9401032735956393	0.8965430924699366	0.8466524587482746	0.7977713994839032	0.8601348383178393
14	0.9537033943467573	0.9218289166613053	0.8849290281382252	0.8468289890135662	0.8592477816975544
15	0.9919817069802634	0.9855961920046116	0.9770091702534148	0.9666218676537932	0.8659586620698194
16	0.9569370597091332	0.9290164519894101	0.8980702082532771	0.8682568770753327	0.8660938511877907
17	0.9786248789817146	0.9602186957706069	0.9360197592977039	0.9084305616325825	0.8684458906569459
18	0.9223114446367479	0.8770570618867215	0.8307616468330447	0.7852973207997480	0.8640696501381460
19	0.9416425925221282	0.9134548252059055	0.8805285436705006	0.8519831496574172	0.8634653251141096
20	0.9673177998488416	0.9513454263159293	0.9375747024636375	0.9247979437089459	0.8663852959995780
480	0.9544918908560924	0.9178789120231019	0.8750499712806408	0.8340254141188145	0.8686907747440842
481	0.9757403189771565	0.9622073908997618	0.9367440524018146	0.9062174218760167	0.8687686308584658
482	0.9308477331795901	0.8869413969329909	0.8383858721374957	0.7925946306322407	0.8686109207130699
483	0.9532233441846450	0.9201888881060101	0.8766106876123528	0.8316221658147255	0.8685344976657593
484	0.9151134776478388	0.8554638955361670	0.7919364527822618	0.7302083950835748	0.8682492892068270
485	0.9795662955911203	0.9516189937461134	0.9133025925847316	0.8744842391229729	0.8682621183218808
486	0.9369475988482806	0.9113635649832024	0.8910566145264066	0.8822540649427032	0.8682908492184328
487	0.9524602115863845	0.9189784554695491	0.87180122037970302	0.8225847221731607	0.8681971891220285
488	0.9567360195017350	0.9235836456009039	0.8818070420846186	0.8427425527311868	0.8681451346508815
489	1.0108681038998930	1.0024453829993905	0.9844003374828131	0.9618233190048368	0.8683363146189509
490	0.9618672640936562	0.9386030289418842	0.9085696833842275	0.8769986715182624	0.8683539568936949
491	0.9336862246598268	0.8952050944364063	0.8532013498915735	0.8170264517910382	0.8682496326963318
492	0.9399421387676299	0.9063243332558951	0.8728164223197123	0.8396338346377501	0.8681915884812028
493	0.9415631164085818	0.9166257187761041	0.88856197711295429	0.8600628947816080	0.8681751336356571
494	0.9778211571974894	0.9638006877910213	0.9456917584422360	0.9258611230878142	0.8682916709880857
495	0.9697188117602353	0.9468225505465561	0.9216009751055934	0.8947788623734229	0.86834509752836207
496	0.9502043714269706	0.9165389846175833	0.8782589413646807	0.8433459319524591	0.8682947725018678
497	0.9598649338986657	0.9300169057199971	0.8973445770933661	0.8681889720864415	0.8682945600512344
498	0.9659910400882985	0.9442615517524653	0.9154982783962132	0.8850486697319836	0.8683281354213361
499	0.9607313858987854	0.9403811282971752	0.9136635495578103	0.8831870458380964	0.8683578532421696
500	0.9560403841219246	0.9247786269881655	0.8860238876961504	0.8470123411155974	0.8683152474295418

[11] analytic zero for 20160704.0000 is 0.8683152474295418

avgpx: 0.8683152474295418 ln: -0.1412004419758045 ln/4: -0.0353001104939511 analytical/4: -0.0353001104939511 diff: 0.0000000000000000

[11] using delrc: 20160704.0000 of -0.0000115910841628: zero: 0.8683152474295418 error: 0.0000000000000000



**Table 5. Calibration of 500 Interest Rate Path Zero Prices**

```

*****
[1] analytic zero for 20100702.0000 is 0.9999761116817766
avg zero : 0.9999753505072757 analytical zero: 0.9999761116817766 diff: -0.0000007611745009 days: 2.0000 delrc:
-0.0000007611929743
avgpx: 0.9999753505072757 ln: -0.0000246497965280 ln/4: -0.0000061624491320 analytical/4: -0.0000059721508885 diff: -0.0000001902982435
avgpx: 0.9999761116817759 ln: -0.0000238886035545 ln/4: -0.0000059721508886 analytical/4: -0.0000059721508885 diff: -0.0000000000000002
[1] using delrc: 20100702.0000 of -0.0000007611929743: zero: 0.9999761116817759 error: -0.0000000000000007

*****
[2] analytic zero for 20100802.0000 is 0.9995887710330013
avg zero : 0.9995168033278677 analytical zero: 0.9995887710330013 diff: -0.0000719677051335 days: 31.0000 delrc:
-0.0000046451551255
avgpx: 0.9995168033278677 ln: -0.0004833134492633 ln/4: -0.0001208283623158 analytical/4: -0.0001028283862046 diff: -0.0000179999761112
avgpx: 0.9995887710330021 ln: -0.0004113135448175 ln/4: -0.0001028283862044 analytical/4: -0.0001028283862046 diff: 0.0000000000000002
[2] using delrc: 20100802.0000 of -0.0000046451551255: zero: 0.9995887710330021 error: 0.0000000000000009

*****
[3] analytic zero for 20100902.0000 is 0.9990555929590419
avg zero : 0.9990448834200901 analytical zero: 0.9990555929590419 diff: -0.0000107095389518 days: 31.0000 delrc:
-0.0000006915948473
avgpx: 0.9990448834200901 ln: -0.0009555729943930 ln/4: -0.0002388932485982 analytical/4: -0.0002362133185651 diff: -0.0000026799300332
avgpx: 0.9990555929590426 ln: -0.0009448532742596 ln/4: -0.0002362133185649 analytical/4: -0.0002362133185651 diff: 0.0000000000000002
[3] using delrc: 20100902.0000 of -0.0000006915948473: zero: 0.9990555929590426 error: 0.0000000000000007

*****
[4] analytic zero for 20101004.0000 is 0.9983598779682215
avg zero : 0.9983168355046199 analytical zero: 0.9983598779682215 diff: -0.0000430424636015 days: 32.0000 delrc:
-0.0000026946314918
avgpx: 0.9983168355046199 ln: -0.0016845826082409 ln/4: -0.0004211456520602 analytical/4: -0.0004103671260931 diff: -0.0000107785259671
avgpx: 0.9983598779682219 ln: -0.0016414685043720 ln/4: -0.0004103671260930 analytical/4: -0.0004103671260931 diff: 0.0000000000000001
[4] using delrc: 20101004.0000 of -0.0000026946314918: zero: 0.9983598779682219 error: 0.0000000000000004

*****
[5] analytic zero for 20110103.0000 is 0.9961368342995802
avg zero : 0.9964663360536166 analytical zero: 0.9961368342995802 diff: 0.0003295017540363 days: 91.0000 delrc:
0.0000072686794498
avgpx: 0.9964663360536166 ln: -0.0035399220839460 ln/4: -0.0008849805209865 analytical/4: -0.0009676617497284 diff: 0.0000826812287419
avgpx: 0.9961368342995804 ln: -0.0038706469989135 ln/4: -0.0009676617497284 analytical/4: -0.0009676617497284 diff: 0.0000000000000001
[5] using delrc: 20110103.0000 of 0.0000072686794498: zero: 0.9961368342995804 error: 0.0000000000000002

*****
[6] analytic zero for 20110704.0000 is 0.9925893171275642
avg zero : 0.9906534495155468 analytical zero: 0.9925893171275642 diff: -0.0019358676120174 days: 182.0000 delrc:
-0.0000214530238818
avgpx: 0.9906534495155468 ln: -0.0093905035750029 ln/4: -0.0023476258937507 analytical/4: -0.0018595696004400 diff: -0.0004880562933107
avgpx: 0.9925893171275632 ln: -0.0074382784017611 ln/4: -0.0018595696004403 analytical/4: -0.0018595696004400 diff: -0.0000000000000003
[6] using delrc: 20110704.0000 of -0.0000214530238818: zero: 0.9925893171275632 error: -0.0000000000000010

*****
[7] analytic zero for 20120702.0000 is 0.9805917372717877
avg zero : 0.9802089702721405 analytical zero: 0.9805917372717877 diff: -0.0003827669996472 days: 364.0000 delrc:
-0.0000021451597822
avgpx: 0.9802089702721405 ln: -0.0199894950772927 ln/4: -0.0049973737693232 analytical/4: -0.0048997689992319 diff: -0.0000976047700913
avgpx: 0.9805917372717866 ln: -0.0195990759969288 ln/4: -0.0048997689992322 analytical/4: -0.0048997689992319 diff: -0.0000000000000003
[7] using delrc: 20120702.0000 of -0.0000021451597822: zero: 0.9805917372717866 error: -0.0000000000000011

```



```

*****
[8] analytic zero for 20130702.0000 is 0.9608245937734201
avg zero : 0.9570348158717541 analytical zero: 0.9608245937734201 diff: -0.0037897779016660 days: 365.0000 delrc:
-0.0000216553238832
avgpx: 0.9570348158717541 ln: -0.0439155079696583 ln/4: -0.0109788769924146 analytical/4: -0.0099908528402430 diff: -0.0009880241521716
avgpx: 0.9608245937734198 ln: -0.0399634113609725 ln/4: -0.0099908528402431 analytical/4: -0.0099908528402430 diff: -0.0000000000000001
[8] using delrc: 20130702.0000 of -0.0000216553238832: zero: 0.9608245937734198 error: -0.0000000000000003

*****
[9] analytic zero for 20140702.0000 is 0.9339424859281182
avg zero : 0.9338375744904124 analytical zero: 0.9339424859281182 diff: -0.0001049114377059 days: 365.0000 delrc:
-0.0000006155512693
avgpx: 0.9338375744904124 ln: -0.0684527589913846 ln/4: -0.0171131897478462 analytical/4: -0.0170851052211828 diff: -0.0000280845266633

avgpx: 0.9339424859281185 ln: -0.0683404208847310 ln/4: -0.0170851052211828 analytical/4: -0.0170851052211828 diff: 0.0000000000000001
[9] using delrc: 20140702.0000 of -0.0000006155512693: zero: 0.9339424859281185 error: 0.0000000000000002

*****
[10] analytic zero for 20150702.0000 is 0.9012265257922865
avg zero : 0.9004314677161949 analytical zero: 0.9012265257922865 diff: -0.0007950580760916 days: 365.0000 delrc:
-0.0000048360819947
avgpx: 0.9004314677161949 ln: -0.1048812219638519 ln/4: -0.0262203054909630 analytical/4: -0.0259996592499545 diff: -0.0002206462410085
avgpx: 0.9012265257922858 ln: -0.1039986369998186 ln/4: -0.0259996592499546 analytical/4: -0.0259996592499545 diff: -0.0000000000000002
[10] using delrc: 20150702.0000 of -0.0000048360819947: zero: 0.9012265257922858 error: -0.0000000000000007

*****
[11] analytic zero for 20160704.0000 is 0.8683152474295418
avg zero : 0.8664653132823088 analytical zero: 0.8683152474295418 diff: -0.0018499341472330 days: 368.0000 delrc:
-0.0000115910841628
avgpx: 0.8664653132823088 ln: -0.1433332014617663 ln/4: -0.0358333003654416 analytical/4: -0.0353001104939511 diff: -0.0005331898714905
avgpx: 0.8683152474295418 ln: -0.1412004419758045 ln/4: -0.0353001104939511 analytical/4: -0.0353001104939511 diff: 0.0000000000000000
[11] using delrc: 20160704.0000 of -0.0000115910841628: zero: 0.8683152474295418 error: 0.0000000000000000

*****
[12] analytic zero for 20170703.0000 is 0.8319240167628068
avg zero : 0.8354074771440445 analytical zero: 0.8319240167628068 diff: 0.0034834603812377 days: 364.0000 delrc:
0.0000229587459036
avgpx: 0.8354074771440445 ln: -0.1798356765831954 ln/4: -0.0449589191457988 analytical/4: -0.0460035420844110 diff: 0.0010446229386121
avgpx: 0.8319240167628068 ln: -0.1840141683376439 ln/4: -0.0460035420844110 analytical/4: -0.0460035420844110 diff: 0.0000000000000000
[12] using delrc: 20170703.0000 of 0.0000229587459036: zero: 0.8319240167628068 error: 0.0000000000000000

*****
[13] analytic zero for 20180702.0000 is 0.7999164674335155
avg zero : 0.7954654602071045 analytical zero: 0.7999164674335155 diff: -0.0044510072264110 days: 364.0000 delrc:
-0.0000306586738778
avgpx: 0.7954654602071045 ln: -0.2288278511197680 ln/4: -0.0572069627799420 analytical/4: -0.0558119931185037 diff: -0.0013949696614383
avgpx: 0.7999164674335159 ln: -0.2232479724740145 ln/4: -0.0558119931185036 analytical/4: -0.0558119931185037 diff: 0.0000000000000001
[13] using delrc: 20180702.0000 of -0.0000306586738778: zero: 0.7999164674335159 error: 0.0000000000000003

*****
[14] analytic zero for 20190702.0000 is 0.7663317877085819
avg zero : 0.7697671515607331 analytical zero: 0.7663317877085819 diff: 0.0034353638521512 days: 365.0000 delrc:
0.0000245087636227
avgpx: 0.7697671515607331 ln: -0.2616672104371565 ln/4: -0.0654168026092891 analytical/4: -0.0665350149495752 diff: 0.001182123402861
avgpx: 0.7663317877085825 ln: -0.2661400597983000 ln/4: -0.0665350149495750 analytical/4: -0.0665350149495752 diff: 0.0000000000000002
[14] using delrc: 20190702.0000 of 0.0000245087636227: zero: 0.7663317877085825 error: 0.0000000000000006

```



\*\*\*\*\*  
[15] analytic zero for 20200702.0000 is 0.7314220252712094  
avg zero : 0.7328159914883742 analytical zero: 0.7314220252712094 diff: 0.0013939662171648 days: 366.0000 delrc:  
0.0000104044614976  
avgpx: 0.7328159914883742 ln: -0.3108606434143972 ln/4: -0.0777151608535993 analytical/4: -0.0781911649671163 diff: 0.0004760041135170  
avgpx: 0.7314220252712091 ln: -0.3127646598684656 ln/4: -0.0781911649671164 analytical/4: -0.0781911649671163 diff: -0.0000000000000001  
[15] using delrc: 20200702.0000 of 0.0000104044614976: zero: 0.7314220252712091 error: -0.0000000000000002

\*\*\*\*\*  
[16] analytic zero for 20220704.0000 is 0.6811701033307189  
avg zero : 0.6789043042265936 analytical zero: 0.6811701033307189 diff: -0.0022657991041253 days: 732.0000 delrc:  
-0.0000091034926141  
avgpx: 0.6789043042265936 ln: -0.3872750976937590 ln/4: -0.0968187744234397 analytical/4: -0.0959858048492511 diff: -0.0008329695741887  
avgpx: 0.6811701033307183 ln: -0.3839432193970053 ln/4: -0.0959858048492513 analytical/4: -0.0959858048492511 diff: -0.0000000000000002  
[16] using delrc: 20220704.0000 of -0.0000091034926141: zero: 0.6811701033307183 error: -0.0000000000000007

\*\*\*\*\*  
[17] analytic zero for 20250702.0000 is 0.6081431365828338  
avg zero : 0.6127025920479947 analytical zero: 0.6081431365828338 diff: 0.0045594554651609 days: 1094.0000 delrc:  
0.0000136551626807  
avgpx: 0.6127025920479947 ln: -0.4898756287257822 ln/4: -0.1224689071814456 analytical/4: -0.1243362506780358 diff: 0.0018673434965903  
avgpx: 0.6081431365828338 ln: -0.4973450027121434 ln/4: -0.1243362506780358 analytical/4: -0.1243362506780358 diff: 0.0000000000000000  
[17] using delrc: 20250702.0000 of 0.0000136551626807: zero: 0.6081431365828338 error: 0.0000000000000000

\*\*\*\*\*  
[18] analytic zero for 20300702.0000 is 0.4928012022237420  
avg zero : 0.4928623832470340 analytical zero: 0.4928012022237420 diff: 0.0000611810232921 days: 1826.0000 delrc:  
0.0000001359712979  
avgpx: 0.4928623832470340 ln: -0.7075252853934300 ln/4: -0.1768813213483575 analytical/4: -0.1769123567971011 diff: 0.0000310354487436  
avgpx: 0.4928012022237414 ln: -0.7076494271884055 ln/4: -0.1769123567971014 analytical/4: -0.1769123567971011 diff: -0.0000000000000003  
[18] using delrc: 20300702.0000 of 0.0000001359712979: zero: 0.4928012022237414 error: -0.0000000000000006

\*\*\*\*\*  
[19] analytic zero for 20400702.0000 is 0.2914580503884611  
avg zero : 0.3262484174798455 analytical zero: 0.2914580503884611 diff: 0.0347903670913844 days: 3653.0000 delrc:  
0.0000617372143595  
avgpx: 0.3262484174798455 ln: -1.1200961709934874 ln/4: -0.2800240427483718 analytical/4: -0.3082147982552747 diff: 0.0281907555069029  
avgpx: 0.2914580503884608 ln: -1.2328591930210995 ln/4: -0.3082147982552749 analytical/4: -0.3082147982552747 diff: -0.0000000000000002  
[19] using delrc: 20400702.0000 of 0.0000617372143595: zero: 0.2914580503884608 error: -0.0000000000000002





Following successful calibration of zero prices at the tick points of each grid, we can begin to construct the monthly interest rate paths which form the basis of the model. Each grid staked out a framework for a given interest rate path structure. In order to generate monthly zero prices and rates with the grid, we must choose either to interpolate in rate space or to interpolate in zero price space. For example, if we assume that we know the grid structure and we wish to find the rate and the zero price for date 't' which falls between the date corresponding to tick 'n' and tick 'n+1', and on tick 'n' and tick 'n+1' we know the corresponding rates and zero prices for the grid we are solving and we wish to determine the rate  $r_t$  by interpolation in rate space, then we would determine the rate by interpolating the rates of the tick points linearly and then we would solve for the corresponding zero price applicable for date 't' :

$$d_n \Rightarrow r_n \Rightarrow Z_n$$

$$d_{n+1} \Rightarrow r_{n+1} \Rightarrow Z_{n+1}$$

$$\Delta t = d_{n+1} - d_n$$

$$frac = (d_t - d_n) / \Delta t$$

$$r_{t(n < t < n+1)} = r_n + frac * (r_{n+1} - r_n)$$

$$Z_t = \exp(-(r_n + r_t) * (frac * \Delta t) / 2) * Z_n$$

Conversely, if the rates were determined from the linear interpolation of the zeros, the formulation would be as follows:

$$d_n \Rightarrow r_n \Rightarrow Z_n$$

$$d_{n+1} \Rightarrow r_{n+1} \Rightarrow Z_{n+1}$$

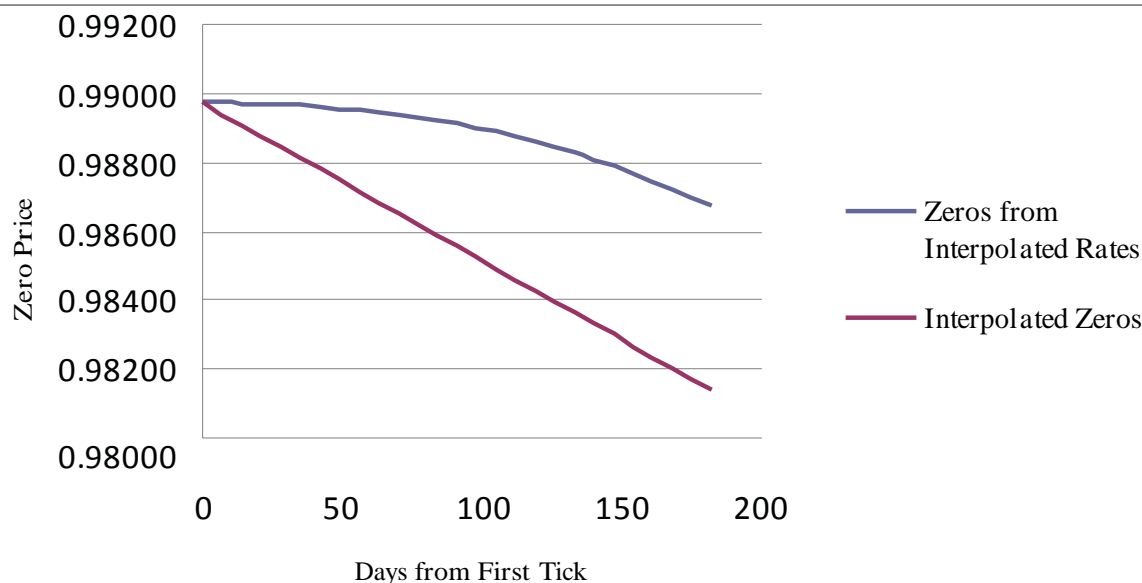
$$\Delta t = d_{n+1} - d_n$$

$$frac = (d_t - d_n) / \Delta t$$

$$Z_t = Z_n^{(1-frac)} * Z_{n+1}^{(frac)}$$

The practical difference of the interpolation methodologies can be seen in the chart below which depicts zero prices between two arbitrary ticks:

Zero Generation Method



Presently, we are linearly interpolating the rates with the appropriate calibration factors and determining the zero prices as in the calculation above.

Proceeding month to month on each grid will result in one 360 month path of short rates for each grid. Similarly, by using the zero prices appropriately, it is possible to produce a monthly path of the 2 year rate, or 10 year rate, or any desired rate process.

Deals that are dependent upon multiple interest rates, such as 3 Month LIBOR and 10 Year LIBOR must be driven in a pathwise fashion such that the spread between the aforementioned interest rates generated by the unified model will not simply be a constant value but will rather depend upon the shape of the simulated path. Naturally, each path has its own vector of short rates and longer term rates.

If we graph all resulting paths of the short rate we will obtain charts that look similar to those in the beginning of this draft.

Regardless of the volatility surface selected, it is possible to determine a set of calibration factors such that the prices for the on-the-run issues computed over all paths will average the price of that same reference on-the-run issue in the market (which after all was the supplied input value). However, securities with embedded options such as caps, floors, and swaptions will only price correctly if a particular volatility surface is input. The greater the volatility, the wider the dispersion of interest rates over time which can be referred to as the “spread of the fan” of rates.

The options market that trades caps, floors, and swaptions quotes volatility in terms of a log-normal input for Black’s model. Fischer Black and Myron Scholes developed the well known Black-Scholes model of option pricing in 1973. While this model was useful for stocks, fixed income options were better priced using a model advanced by Black which was a form of the Black-Scholes model whereby the spot price was replaced by the future or forward price. Given that in the futures market, the future price is known, the Black model was a better fit. The underlying assumption is that the path of the future price at the option expiry follows a lognormal process.

Quotes of volatility for Black’s model are then transformed by each modeler according to the characteristics of his/her particular model and tested for accuracy in predicting option pricing vs. the market. While it is formulaicly possible to convert quoted volatilities for Black’s model into normal volatilities, it is not useful for the scope of the DBRS unified interest rate model. This is due to the flexibility realised by using





the quoted Black's volatilities available for many markets in conjunction with one overall stress factor thereby allowing rapid determination of the interest rate fan for markets having full pricing structures.

Once the volatility surface has been downloaded from Bloomberg and the appropriate stress factor has been applied, the system can be calibrated and the interest rate fan can be produced for all desired maturity instruments monthly for a term of 30 years. For instance, it is possible to generate 500 paths of 10 Year LIBOR where each path specifies 10 Year LIBOR monthly over a course of 30 years.

Following the generation of the fan for a particular index maturity, all possible rate values for every month are extracted and sorted from highest rate to the lowest. Based upon the confidence interval required for a given rating, the appropriate rate for that particular month will be recorded in the corresponding output matrix using the formula  $rank = p(n + 1)$  where  $p$  refers to the confidence interval percentage and  $n$  is the number of ordered items considered while the  $rank$  denotes the position in the ordered list.

Rating	Hypothetical Confidence Interval
AAA	99.90%
AA	99.75%
A	99.47%
BBB	97.82%
BB	87.50%
B	77.49%
CCC	71.92%
C	0.00%

The actual confidence interval is time-dependent based upon the DBRS idealised default table in Appendix 1. For illustrative purposes, the above table is being utilised to depict the methodology.

The precise determination of a stress multiple to be applied to the volatility input is, in fact, more of an art than a science. This value, which implicitly incorporates a correction for lognormal/normal conversion, is entirely subjective. The advantage of this implementation is that the entire system of projected rates is consistent within a given currency's swap market and from currency to currency. Furthermore, the use of a single multiplier over all currencies certainly simplifies the procedural aspects and allows focus on the entire spectrum of rates generated without the cost of manipulating many variables for each currency. Finally, it is certainly possible to specify a multiplier for each currency under consideration.

## EXTENSION TO OTHER MARKETS

A key consideration in the design of the DBRS Unified Interest Rate Model was that the same algorithmic approach should be capable of curve generation for each market being considered. This design goal has indeed been achieved as will be explained presently.

Markets to be Considered
Canadian Dollar Swaps – CDN
Swiss Frank Swaps – CHF
Danish Krone Swaps- DKK
Euro Swaps – EUR
Pound Sterling Swaps – GBP
Japanese Yen Swaps – JPY
Norwegian Krone Swaps – NOK
Swedish Krone Swaps - SEK
US Dollar Swaps – USD



Before any curves can be constructed, it is necessary to compile history for each enumerated market. Facilitating this requirement, the Bloomberg was utilised to capture historical swap rates and cap/floor implied volatilities from 1/1/2000 to the present. This historical database allows the software controlling the interest rate model to generate curves for any date for any market for which the data exists. It was discovered that there is no regularly quoted market in Canadian Dollar implied cap/floor volatility and therefore the systematic approach will not be able to be extended to this market without a critical assumption of CDN volatility.

As desired, the database can be updated via Bloomberg's history tool and presented to the interest rate model such that curves can be generated for any market based upon the previous nights closing levels.

For completeness, it is beneficial to list the Bloomberg tickers for each component of the historical data pull.

<b>Bloomberg Ticker</b>	<b>Canadian Dollar Swap Rates</b>
CDOR01 Equity	CDN 1 month swap rate
CDOR03 Equity	CDN 3 month swap rate
CDOR06 Equity	CDN 6 month swap rate
CDOR12 Equity	CDN 1 year swap rate
CDSW2 Equity	CDN 2 year swap rate
CDSW3 Equity	CDN 3 year swap rate
CDSW4 Equity	CDN 4 year swap rate
CDSW5 Equity	CDN 5 year swap rate
CDSW6 Equity	CDN 6 year swap rate
CDSW7 Equity	CDN 7 year swap rate
CDSW8 Equity	CDN 8 year swap rate
CDSW9 Equity	CDN 9 year swap rate
CDSW10 Equity	CDN 10 year swap rate
CDSW15 Equity	CDN 15 year swap rate
CDSW20 Equity	CDN 20 year swap rate
CDSW30 Equity	CDN 30 year swap rate

<b>Bloomberg Ticker</b>	<b>Swiss Franc Swap Rates</b>
SF0001M Index	CHF 1 month swap rate
SF0003M Index	CHF 3 month swap rate
SF0006M Index	CHF 6 month swap rate
SF0012M Index	CHF 1 year swap rate
SFSW2 Index	CHF 2 year swap rate
SFSW3 Index	CHF 3 year swap rate
SFSW4 Index	CHF 4 year swap rate
SFSW5 Index	CHF 5 year swap rate
SFSW6 Index	CHF 6 year swap rate
SFSW7 Index	CHF 7 year swap rate
SFSW8 Index	CHF 8 year swap rate
SFSW9 Index	CHF 9 year swap rate
SFSW10 Index	CHF 10 year swap rate
SFSW15 Index	CHF 15 year swap rate
SFSW20 Index	CHF 20 year swap rate
SFSW30 Index	CHF 30 year swap rate



Bloomberg Ticker	Swiss Franc Implied Cap Vol
SFCFA1 Equity CHF	1 Year implied cap/floor volatility
SFCFA2 Equity CHF	2 Year implied cap/floor volatility
SFCFA3 Equity CHF	3 Year implied cap/floor volatility
SFCFA4 Equity CHF	4 Year implied cap/floor volatility
SFCFA5 Equity CHF	5 Year implied cap/floor volatility
SFCFA6 Equity CHF	6 Year implied cap/floor volatility
SFCFA7 Equity CHF	7 Year implied cap/floor volatility
SFCFA8 Equity CHF	8 Year implied cap/floor volatility
SFCFA9 Equity CHF	9 Year implied cap/floor volatility
SFCFA10 Equity CHF	10 Year implied cap/floor volatility
SFCFA15 Equity CHF	15 Year implied cap/floor volatility
SFCFA20 Equity CHF	20 Year implied cap/floor volatility

Bloomberg Ticker	Danish Krone Swap Rates
CIBO01M Equity	DKK 1 month swap rate
CIBO03M Equity	DKK 3 month swap rate
CIBO06M Equity	DKK 6 month swap rate
DKSW1 Equity	DKK 1 year swap rate
DKSW2 Equity	DKK 2 year swap rate
DKSW3 Equity	DKK 3 year swap rate
DKSW4 Equity	DKK 4 year swap rate
DKSW5 Equity	DKK 5 year swap rate
DKSW6 Equity	DKK 6 year swap rate
DKSW7 Equity	DKK 7 year swap rate
DKSW8 Equity	DKK 8 year swap rate
DKSW9 Equity	DKK 9 year swap rate
DKSW10 Equity	DKK 10 year swap rate
DKSW15 Equity	DKK 15 year swap rate
DKSW20 Equity	DKK 20 year swap rate
DKSW30 Equity	DKK 30 year swap rate

Bloomberg Ticker	Danish Krone Implied Cap Vol
DKCFA1 Equity DKK	1 Year implied cap/floor volatility
DKCFA2 Equity DKK	2 Year implied cap/floor volatility
DKCFA3 Equity DKK	3 Year implied cap/floor volatility
DKCFA4 Equity DKK	4 Year implied cap/floor volatility
DKCFA5 Equity DKK	5 Year implied cap/floor volatility
DKCFA6 Equity DKK	6 Year implied cap/floor volatility
DKCFA7 Equity DKK	7 Year implied cap/floor volatility
DKCFA8 Equity DKK	8 Year implied cap/floor volatility
DKCFA9 Equity DKK	9 Year implied cap/floor volatility
DKCFA10 Equity DKK	10 Year implied cap/floor volatility



Bloomberg Ticker	Euro Swap Rates
EUR001M Index	EUR 1 month swap rate
EUR003M Index	EUR 3 month swap rate
EUR006M Index	EUR 6 month swap rate
EUR012M Index	EUR 1 year swap rate
EUSA2 Index	EUR 2 year swap rate
EUSA3 Index	EUR 3 year swap rate
EUSA4 Index	EUR 4 year swap rate
EUSA5 Index	EUR 5 year swap rate
EUSA6 Index	EUR 6 year swap rate
EUSA7 Index	EUR 7 year swap rate
EUSA8 Index	EUR 8 year swap rate
EUSA9 Index	EUR 9 year swap rate
EUSA10 Index	EUR 10 year swap rate
EUSA15 Index	EUR 15 year swap rate
EUSA20 Index	EUR 20 year swap rate
EUSA30 Index	EUR 30 year swap rate

Bloomberg Ticker	Euro Implied Cap Vol
EUCV1 Equity EUR	1 Year implied cap/floor volatility
EUCV2 Equity EUR	2 Year implied cap/floor volatility
EUCV3 Equity EUR	3 Year implied cap/floor volatility
EUCV4 Equity EUR	4 Year implied cap/floor volatility
EUCV5 Equity EUR	5 Year implied cap/floor volatility
EUCV6 Equity EUR	6 Year implied cap/floor volatility
EUCV7 Equity EUR	7 Year implied cap/floor volatility
EUCV8 Equity EUR	8 Year implied cap/floor volatility
EUCV9 Equity EUR	9 Year implied cap/floor volatility
EUCV10 Equity EUR	10 Year implied cap/floor volatility
EUCV15 Equity EUR	15 Year implied cap/floor volatility
EUCV20 Equity EUR	20 Year implied cap/floor volatility
EUCV30 Equity EUR	30 Year implied cap/floor volatility



Bloomberg Ticker	British Pound Swap Rates
BP0001M Equity	GBP 1 month swap rate
BP0003M Equity	GBP 3 month swap rate
BP0006M Equity	GBP 6 month swap rate
BP0012M Equity	GBP 1 year swap rate
BPSW2 Equity	GBP 2 year swap rate
BPSW3 Equity	GBP 3 year swap rate
BPSW4 Equity	GBP 4 year swap rate
BPSW5 Equity	GBP 5 year swap rate
BPSW6 Equity	GBP 6 year swap rate
BPSW7 Equity	GBP 7 year swap rate
BPSW8 Equity	GBP 8 year swap rate
BPSW9 Equity	GBP 9 year swap rate
BPSW10 Equity	GBP 10 year swap rate
BPSW15 Equity	GBP 15 year swap rate
BPSW20 Equity	GBP 20 year swap rate
BPSW30 Equity	GBP 30 year swap rate

Bloomberg Ticker	British Pound Implied Cap Vol
BPCFA1 Equity GBP	1 Year implied cap/floor volatility
BPCFA2 Equity GBP	2 Year implied cap/floor volatility
BPCFA3 Equity GBP	3 Year implied cap/floor volatility
BPCFA4 Equity GBP	4 Year implied cap/floor volatility
BPCFA5 Equity GBP	5 Year implied cap/floor volatility
BPCFA6 Equity GBP	6 Year implied cap/floor volatility
BPCFA7 Equity GBP	7 Year implied cap/floor volatility
BPCFA8 Equity GBP	8 Year implied cap/floor volatility
BPCFA9 Equity GBP	9 Year implied cap/floor volatility
BPCFA10 Equity GBP	10 Year implied cap/floor volatility
BPCFA15 Equity GBP	15 Year implied cap/floor volatility
BPCFA20 Equity GBP	20 Year implied cap/floor volatility



Bloomberg Ticker	Japanese Yen Swap Rates
JY0001M Equity	JPY 1 month swap rate
JY0003M Equity	JPY 3 month swap rate
JY0006M Equity	JPY 6 month swap rate
JY0012M Equity	JPY 1 year swap rate
BRTM2YR Equity	JPY 2 year swap rate
BRTM3YR Equity	JPY 3 year swap rate
BRTM4YR Equity	JPY 4 year swap rate
BRTM5YR Equity	JPY 5 year swap rate
BRTM6YR Equity	JPY 6 year swap rate
BRTM7YR Equity	JPY 7 year swap rate
BRTM8YR Equity	JPY 8 year swap rate
BRTM9YR Equity	JPY 9 year swap rate
BRTM10YR Equity	JPY 10 year swap rate
BRTM15YR Equity	JPY 15 year swap rate
BRTM20YR Equity	JPY 20 year swap rate
BRTM30YR Equity	JPY 30 year swap rate

Bloomberg Ticker	Japanese Yen Implied Cap Vol
JYCFA1 Equity JPY	1 Year implied cap/floor volatility
JYCFA2 Equity JPY	2 Year implied cap/floor volatility
JYCFA3 Equity JPY	3 Year implied cap/floor volatility
JYCFA4 Equity JPY	4 Year implied cap/floor volatility
JYCFA5 Equity JPY	5 Year implied cap/floor volatility
JYCFA6 Equity JPY	6 Year implied cap/floor volatility
JYCFA7 Equity JPY	7 Year implied cap/floor volatility
JYCFA8 Equity JPY	8 Year implied cap/floor volatility
JYCFA9 Equity JPY	9 Year implied cap/floor volatility
JYCFA10 Equity JPY	10 Year implied cap/floor volatility
JYCFA15 Equity JPY	15 Year implied cap/floor volatility
JYCFA20 Equity JPY	20 Year implied cap/floor volatility



Bloomberg Ticker	Norwegian Krone Swap Rates
NIBOR1M Equity	NOK 1 month swap rate
NIBOR3M Equity	NOK 3 month swap rate
NIBOR6M Equity	NOK 6 month swap rate
NKSW1 Equity	NOK 1 year swap rate
NKSW2 Equity	NOK 2 year swap rate
NKSW3 Equity	NOK 3 year swap rate
NKSW4 Equity	NOK 4 year swap rate
NKSW5 Equity	NOK 5 year swap rate
NKSW6 Equity	NOK 6 year swap rate
NKSW7 Equity	NOK 7 year swap rate
NKSW8 Equity	NOK 8 year swap rate
NKSW9 Equity	NOK 9 year swap rate
NKSW10 Equity	NOK 10 year swap rate
NKSW15 Equity	NOK 15 year swap rate
NKSW20 Equity	NOK 20 year swap rate
NKSW30 Equity	NOK 30 year swap rate

Bloomberg Ticker	Norwegian Krone Implied Cap Vol
NKCFA1 Equity NOK	1 Year implied cap/floor volatility
NKCFA2 Equity NOK	2 Year implied cap/floor volatility
NKCFA3 Equity NOK	3 Year implied cap/floor volatility
NKCFA4 Equity NOK	4 Year implied cap/floor volatility
NKCFA5 Equity NOK	5 Year implied cap/floor volatility
NKCFA6 Equity NOK	6 Year implied cap/floor volatility
NKCFA7 Equity NOK	7 Year implied cap/floor volatility
NKCFA8 Equity NOK	8 Year implied cap/floor volatility
NKCFA9 Equity NOK	9 Year implied cap/floor volatility
NKCFA10 Equity NOK	10 Year implied cap/floor volatility

Bloomberg Ticker	Swedish Krone Swap Rates
AIBO1M Equity	SEK 1 month swap rate
AIBO3M Equity	SEK 3 month swap rate
AIBO6M Equity	SEK 6 month swap rate
AIBO12M Equity	SEK 1 year swap rate
DGSW2 Equity	SEK 2 year swap rate
DGSW3 Equity	SEK 3 year swap rate
DGSW4 Equity	SEK 4 year swap rate
DGSW5 Equity	SEK 5 year swap rate
DGSW6 Equity	SEK 6 year swap rate
DGSW7 Equity	SEK 7 year swap rate
DGSW8 Equity	SEK 8 year swap rate
DGSW9 Equity	SEK 9 year swap rate
DGSW10 Equity	SEK 10 year swap rate
DGSW15 Equity	SEK 15 year swap rate
DGSW20 Equity	SEK 20 year swap rate
DGSW30 Equity	SEK 30 year swap rate



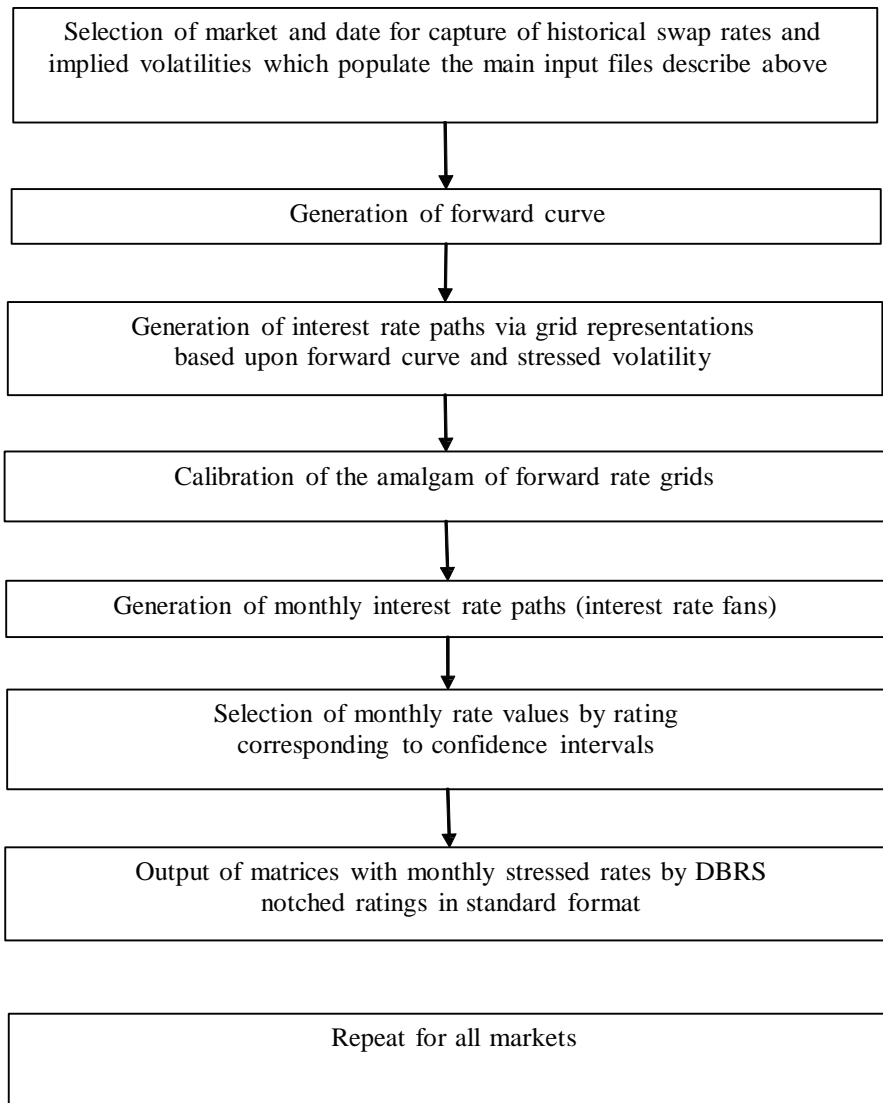
Bloomberg Ticker	Swedish Krone Implied Cap Vol
SKCFA1 Equity SEK	1 Year implied cap/floor volatility
SKCFA2 Equity SEK	2 Year implied cap/floor volatility
SKCFA3 Equity SEK	3 Year implied cap/floor volatility
SKCFA4 Equity SEK	4 Year implied cap/floor volatility
SKCFA5 Equity SEK	5 Year implied cap/floor volatility
SKCFA6 Equity SEK	6 Year implied cap/floor volatility
SKCFA7 Equity SEK	7 Year implied cap/floor volatility
SKCFA8 Equity SEK	8 Year implied cap/floor volatility
SKCFA9 Equity SEK	9 Year implied cap/floor volatility
SKCFA10 Equity SEK	10 Year implied cap/floor volatility

Bloomberg Ticker	US Dollar Swap Rates
US0001M Equity	USD 1 month swap rate
US0003M Equity	USD 3 month swap rate
US0006M Equity	USD 6 month swap rate
US0012M Equity	USD 1 year swap rate
USSF2 Equity	USD 2 year swap rate
USSF3 Equity	USD 3 year swap rate
USSF4 Equity	USD 4 year swap rate
USSF5 Equity	USD 5 year swap rate
USSF6 Equity	USD 6 year swap rate
USSF7 Equity	USD 7 year swap rate
USSF8 Equity	USD 8 year swap rate
USSF9 Equity	USD 9 year swap rate
USSF10 Equity	USD 10 year swap rate
USSF15 Equity	USD 15 year swap rate
USSF20 Equity	USD 20 year swap rate
USSF30 Equity	USD 30 year swap rate

Bloomberg Ticker	US Dollar Implied Cap Vol
USCV1 Equity USD	1 Year implied cap/floor volatility
USCV2 Equity USD	2 Year implied cap/floor volatility
USCV3 Equity USD	3 Year implied cap/floor volatility
USCV4 Equity USD	4 Year implied cap/floor volatility
USCV5 Equity USD	5 Year implied cap/floor volatility
USCV6 Equity USD	6 Year implied cap/floor volatility
USCV7 Equity USD	7 Year implied cap/floor volatility
USCV8 Equity USD	8 Year implied cap/floor volatility
USCV9 Equity USD	9 Year implied cap/floor volatility
USCV10 Equity USD	10 Year implied cap/floor volatility
USCV15 Equity USD	15 Year implied cap/floor volatility

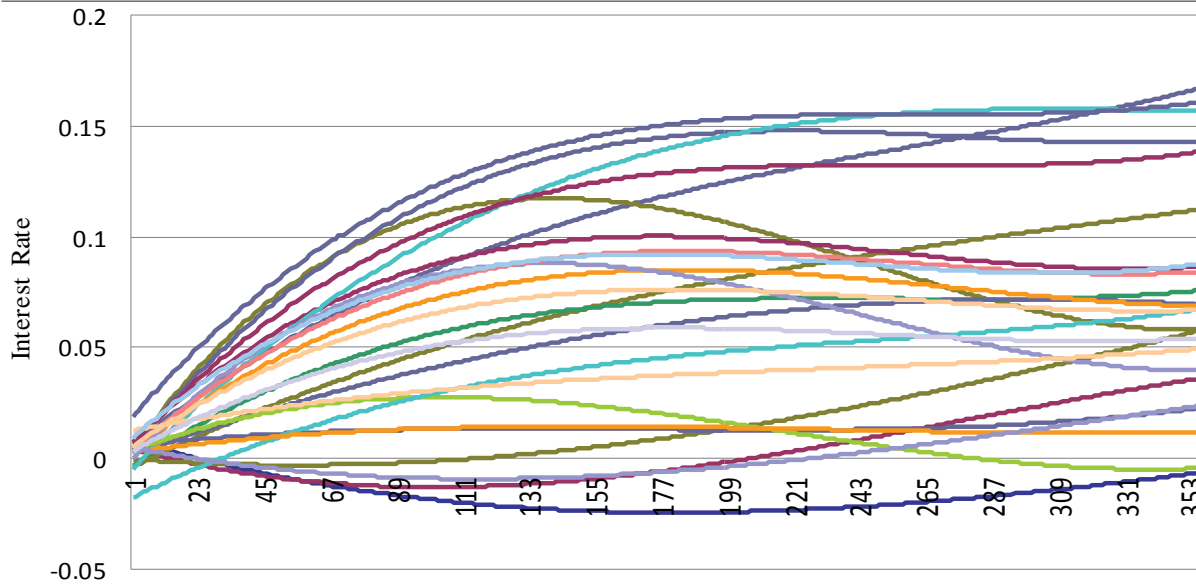
Because the methodology is standardised and uniform, a simple PERL script directs the various programs to operate in the correct sequence and in short order stressed interest rate curve matrices are generated for each market. A pictorial outline illustrates the functionality of the PERL script.



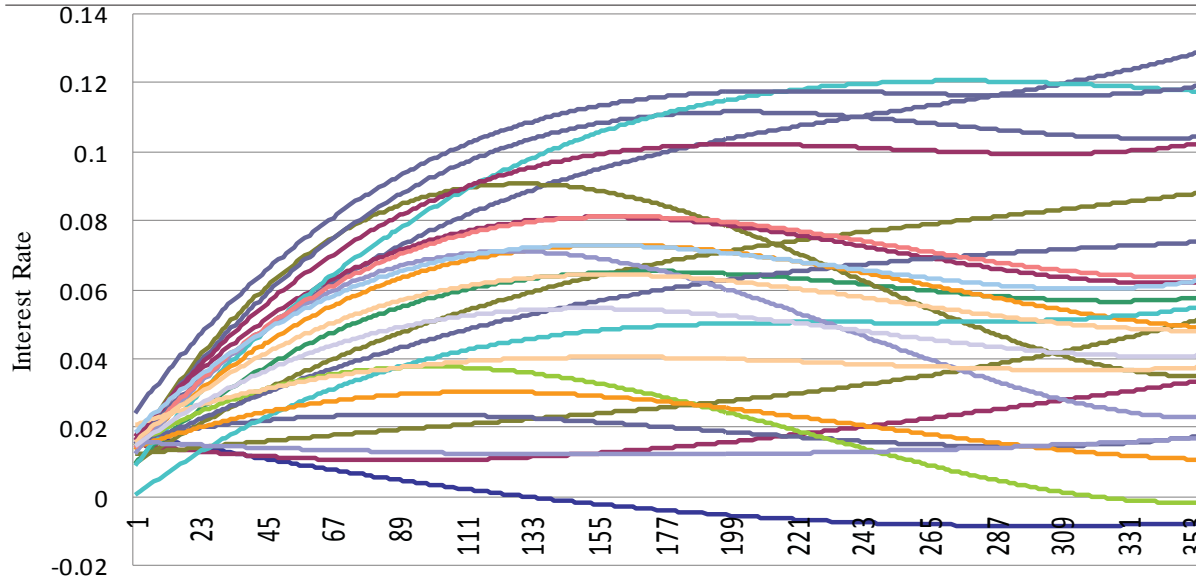




CHF Interest Rate Fan

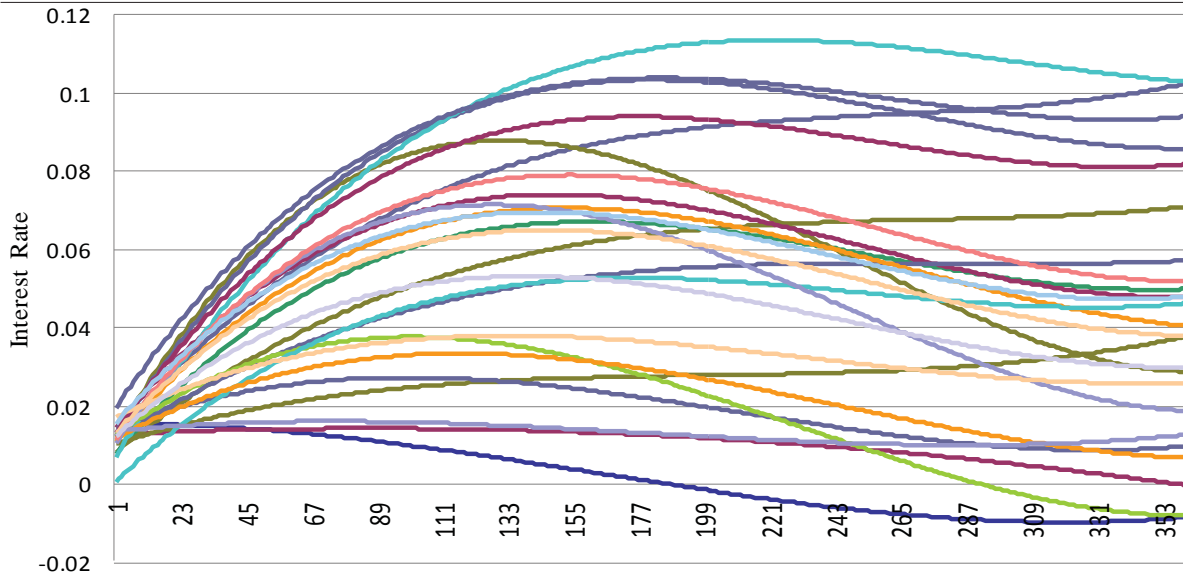


DKK Interest Rate Fan

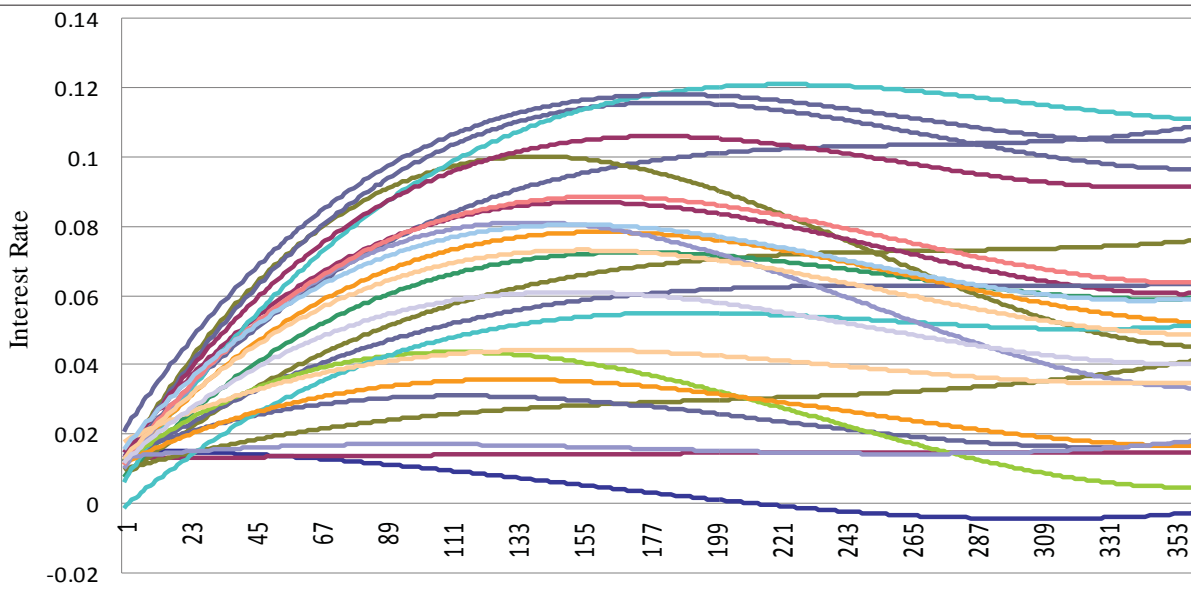




EUR Interest Rate Fan

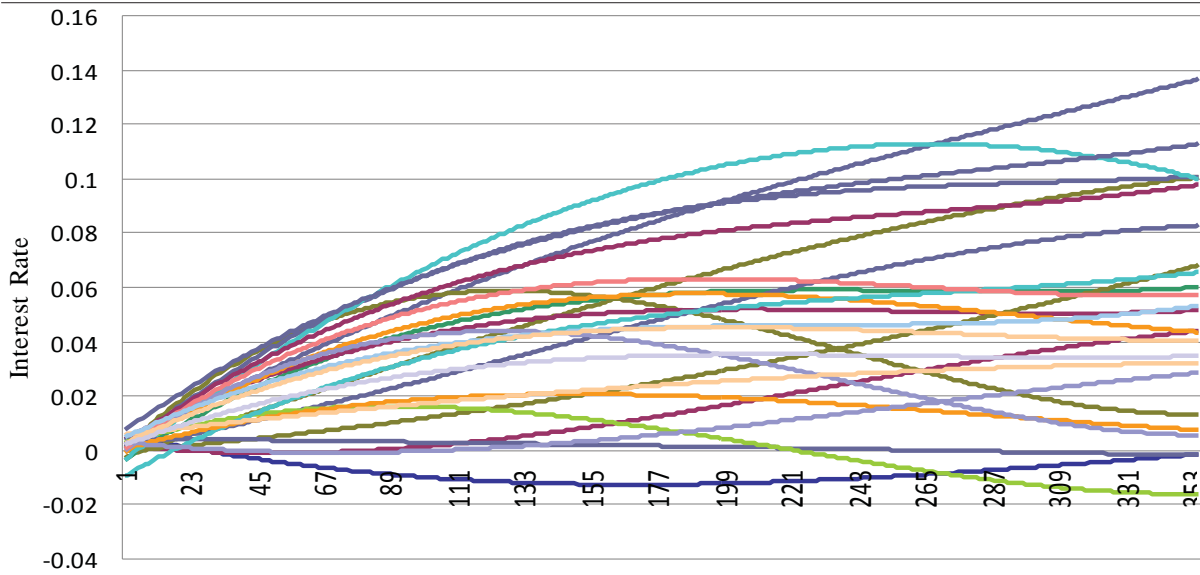


GBP Interest Rate Fan

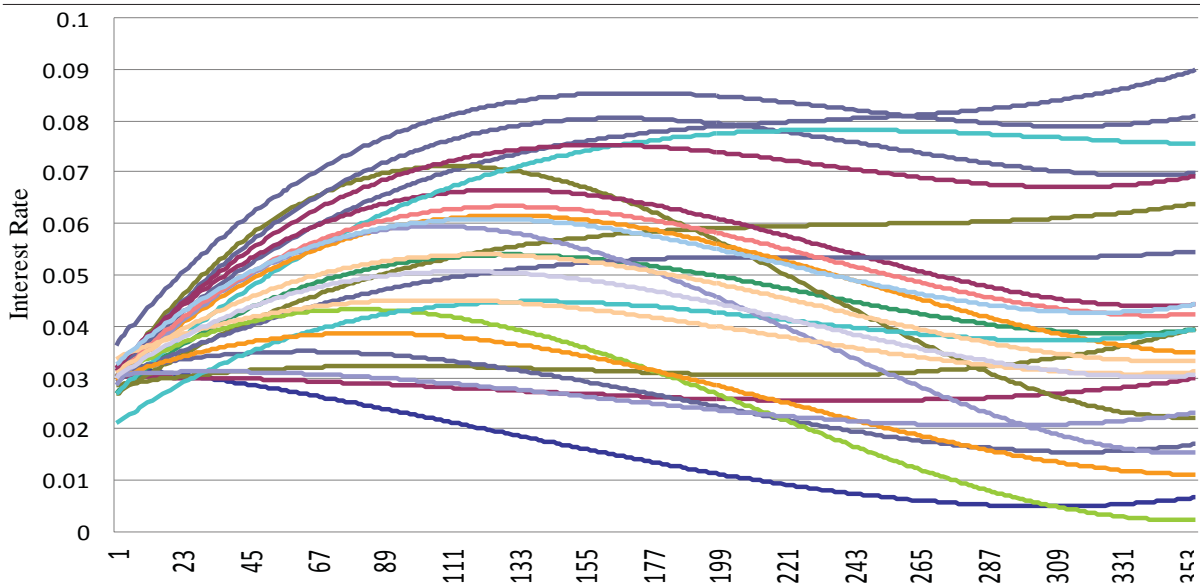




JPY Interest Rate Fan

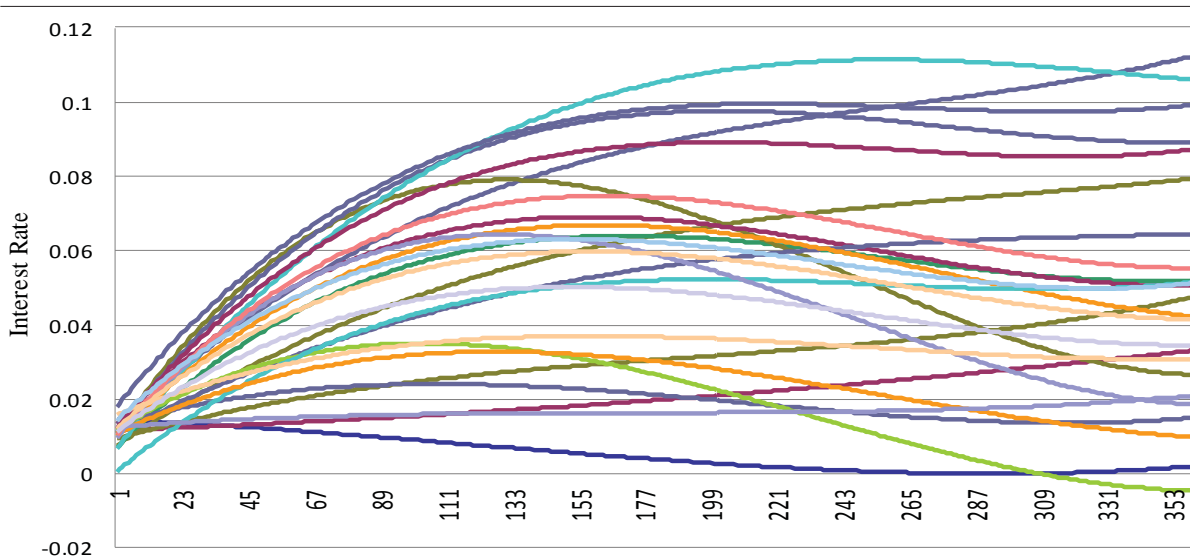


NOK Interest Rate Fan

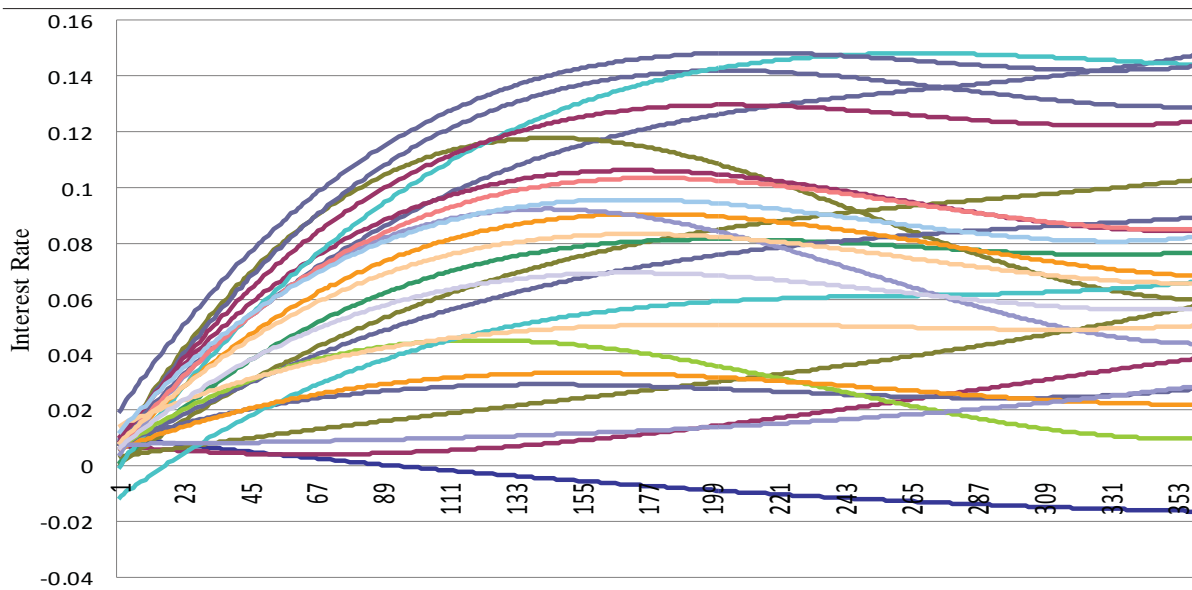




SEK Interest Rate Fan



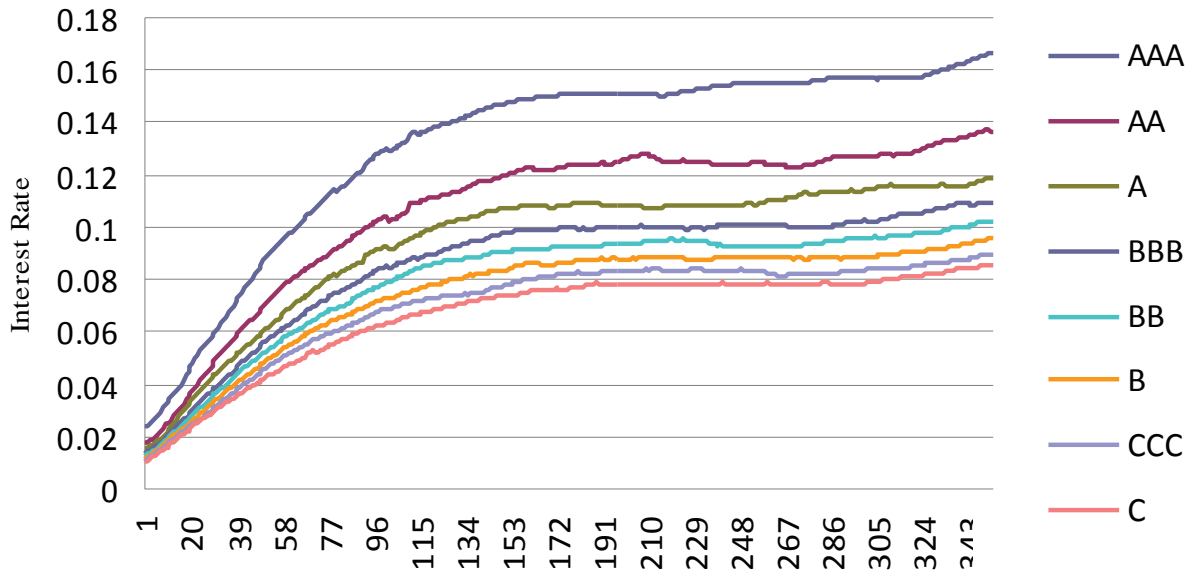
USD Interest Rate Fan



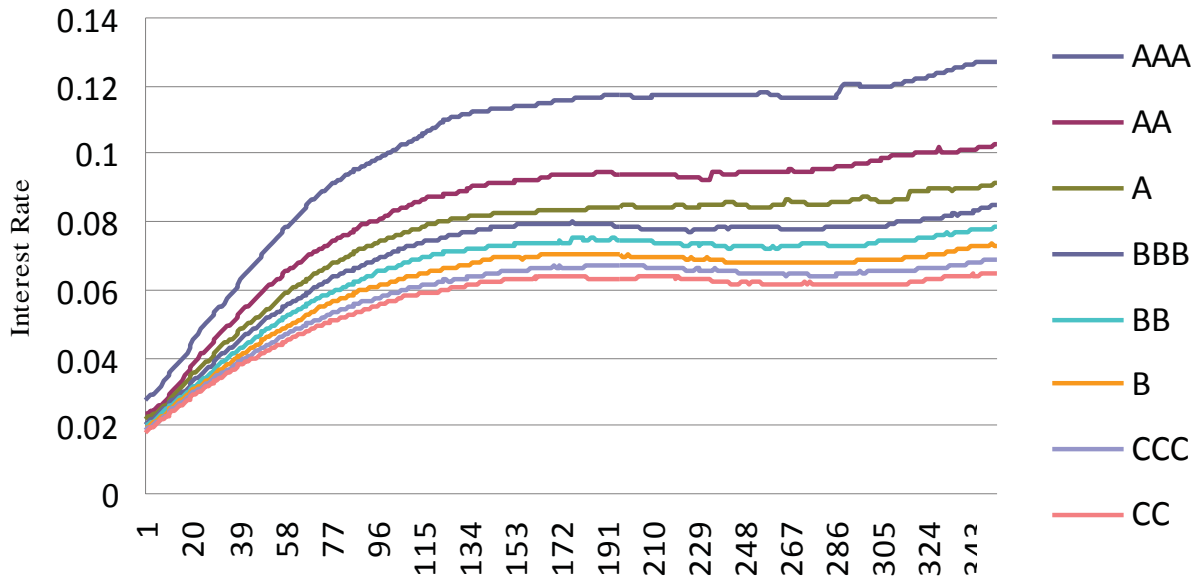
The interest rate fans presented here contain 25 interest rate paths as presenting the full complement of 1000 paths would not be illustrative. It is important to note that all fans were generated at a multiple of 1.75 to the periodic volatilities captured from the Bloomberg history tool. The greater the multiple, the wider the fan, and consequently, the higher the stressed rates in the output matrix.



CHF Stressed Rates - 1.75 Multiplier

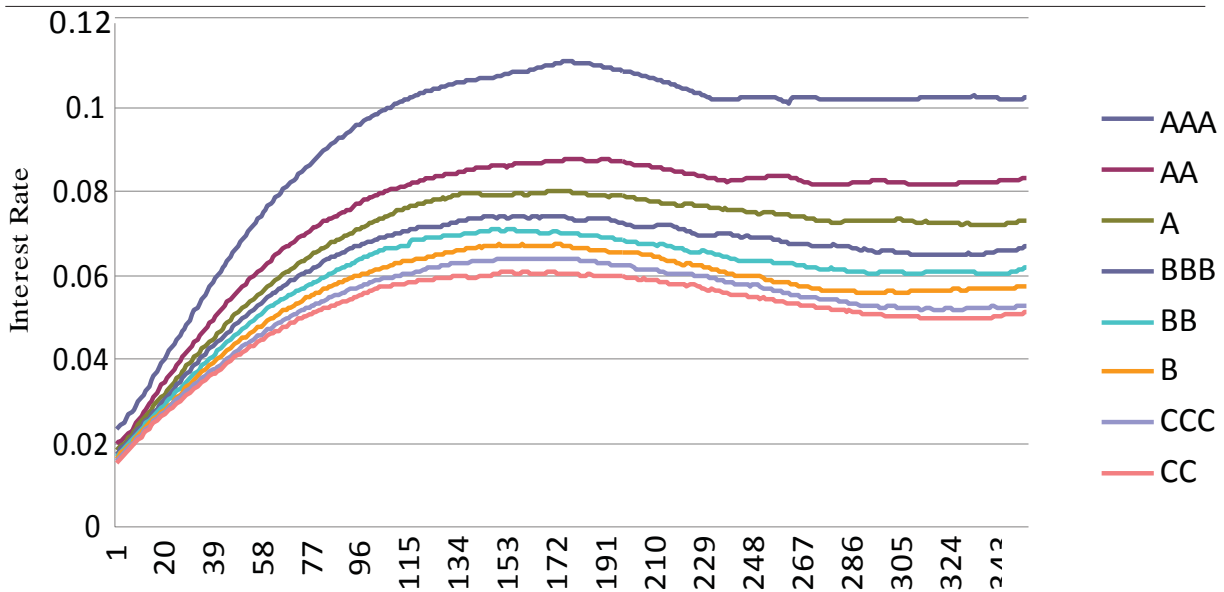


DKK Stressed Rates - 1.75 Multiplier

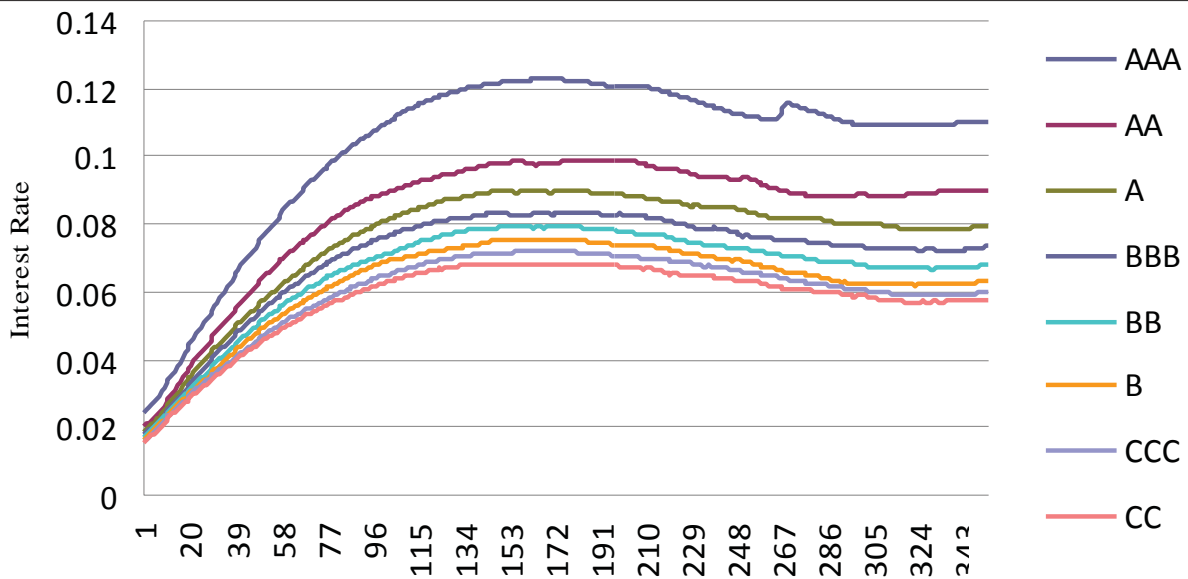




EUR Stressed Rates - 1.75 Multiplier

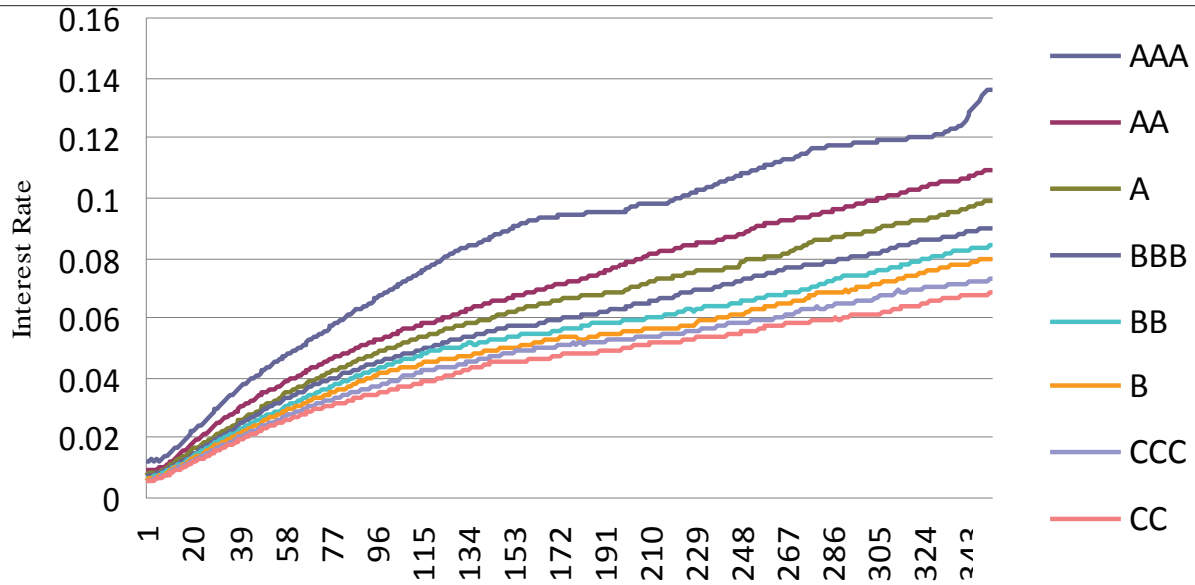


GBP Stressed Rates - 1.75 Multiplier

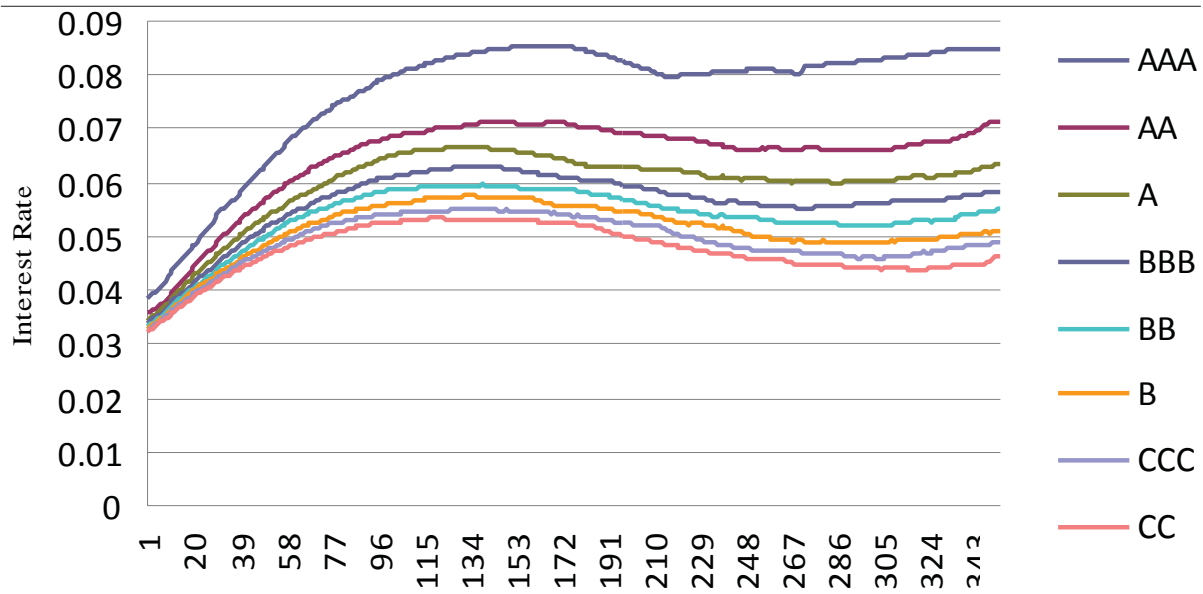




JPY Stressed Rates - 1.75 Multiplier



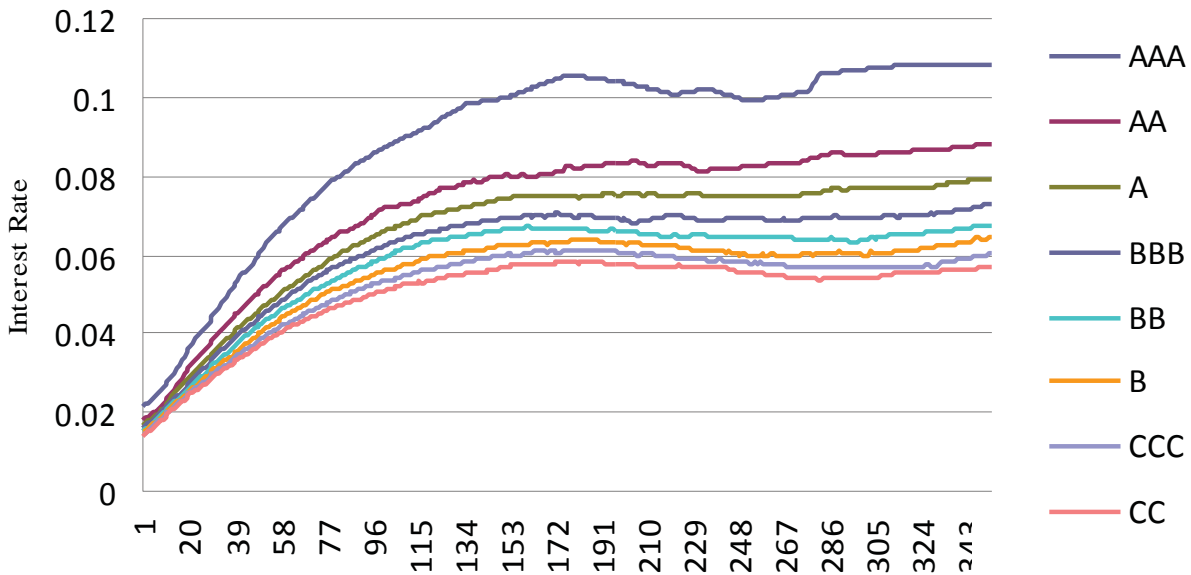
NOK Stressed Rates - 1.75 Multiplier



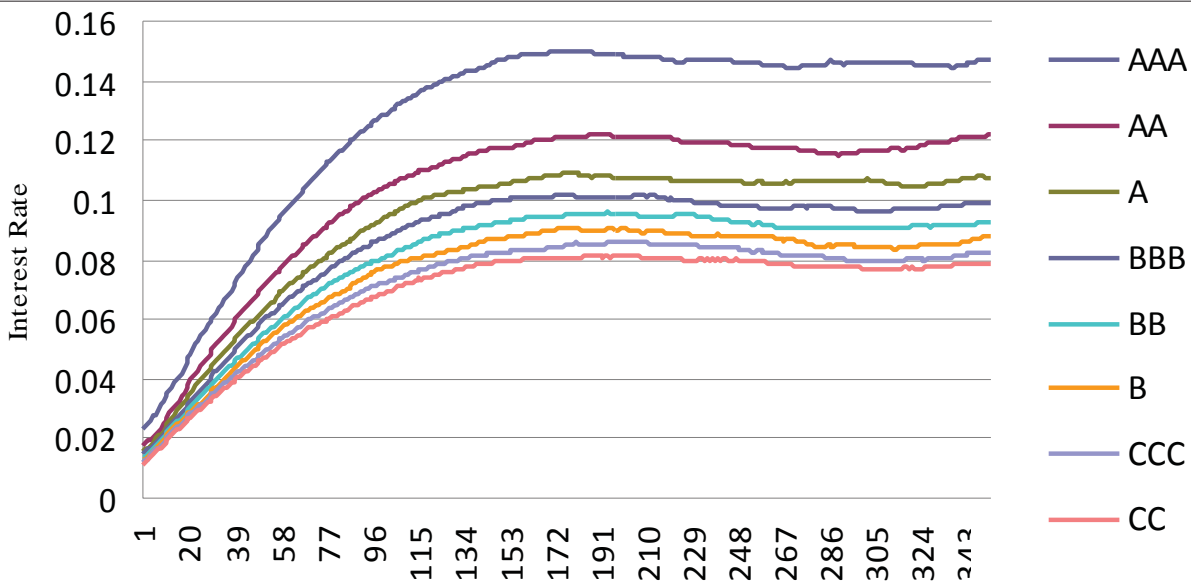




SEK Stressed Rates - 1.75 Multiplier



USD Stressed Rates - 1.75 Multiplier



It is interesting to note the different shapes of the stressed rate curves. Because the only stress control is the one single multiplier, the basic shape of the curvature of the rates evolves solely from the representation of the volatility component. The volatility utilised for these charts was the volatility associated with the market close on 8/9/2010. It is possible to use a different measure of volatility such as historical 90 day volatility. The programming does not depend on the origin of the volatility input as long as a one dimensional surface (a succession of dates and associated volatilities) is presented to the model and that the volatility surface is determined in a consistent fashion across all markets.



## PROCEDURE FOR INCLUSION OF NEGATIVE RATES

As explained previously, the normal model includes the capability to generate negative interest rates along the projected paths. Previously, DBRS included a floor of 0.0 to restrict rate curves to positive values. Concurrent with the modification of volatility, DBRS is also lowering the rate floors based upon tenor with regard to the EUR market as follows:

Tenor in months	Interest Rate Floor
0 – 59 months	-0.50%
60 – 119 months	-0.40%
120-360 months	-0.05%



## VOLATILITY CHOICE: SPOT IMPLIED VERSUS HISTORICAL

Use of the daily spot implied volatility values will lead to whipsaw-like changes in the stressed interest rate fans. Converting to usage of a longer moving average will tend to dampen out such rapid shifts and instead will lead to more stability in the stressed rates being provided to our clients. The UIRM incorporates the 180 day moving average volatility to stabilise the interest rate stress matrices provided to our clients.

Volatility Representations



Recently, it has been observed that volatility measured in the EUR market specifically has increased dramatically, which has caused the interest rate fans to widen and therefore UIRM produced upward-stressed interest rates have increased to unrealistic and hence unusable levels. DBRS has therefore modified its selection process for volatility to adopt the historical volatilities associated with the 180-day period terminating on June 2, 2014, on a going-forward basis. DBRS's analysis of the impact of this change leads DBRS to believe that it is noteworthy, but will have no impact on any currently rated interest rate assets. DBRS is monitoring the developments in the EUR market and further refinements in the UIRM model are currently in development.

## CORRELATION METHODOLOGY TO INCORPORATE PRIME, COFI, CP ETC.

Naturally there are markets that do not have a structure engendered by nodes corresponding to a time dependent swap curve. Markets like Prime, COFI, and CP are single point markets. While this type of representation is not handled by the UIRM, it is possible to produce a stressed curve for each of these markets via regressions of historical values of Prime, COFI, CP, etc. to USD 1M LIBOR and/or other indices. With the correlation factors in hand, the short duration market curve can be generated following which the correlation factors are applied to the selected index to yield the appropriate single point market curve.

The 90 day commercial paper rate was approximated utilising linear equation based upon 1M USD LIBOR.

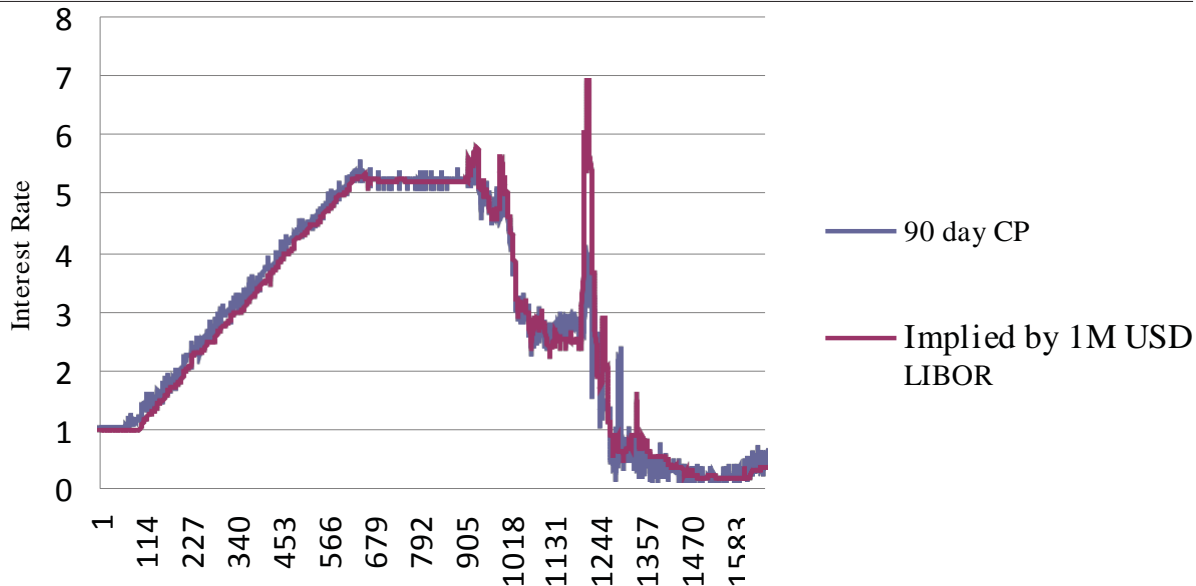
$$CP = L - 0.10152$$



Where  $CP$  represents the 90 day commercial paper rate and  $L$  is equal to the USD 1M LIBOR rate. Although the spike observed at approximately 1300 days does not track the CP market, the simplicity of the fitting equation and the apparent agreement for other values of 1M LIBOR indicate a satisfactory result.

The goodness of fit can be observed in the following chart:

90 Day CP Implied by 1 Month USD LIBOR



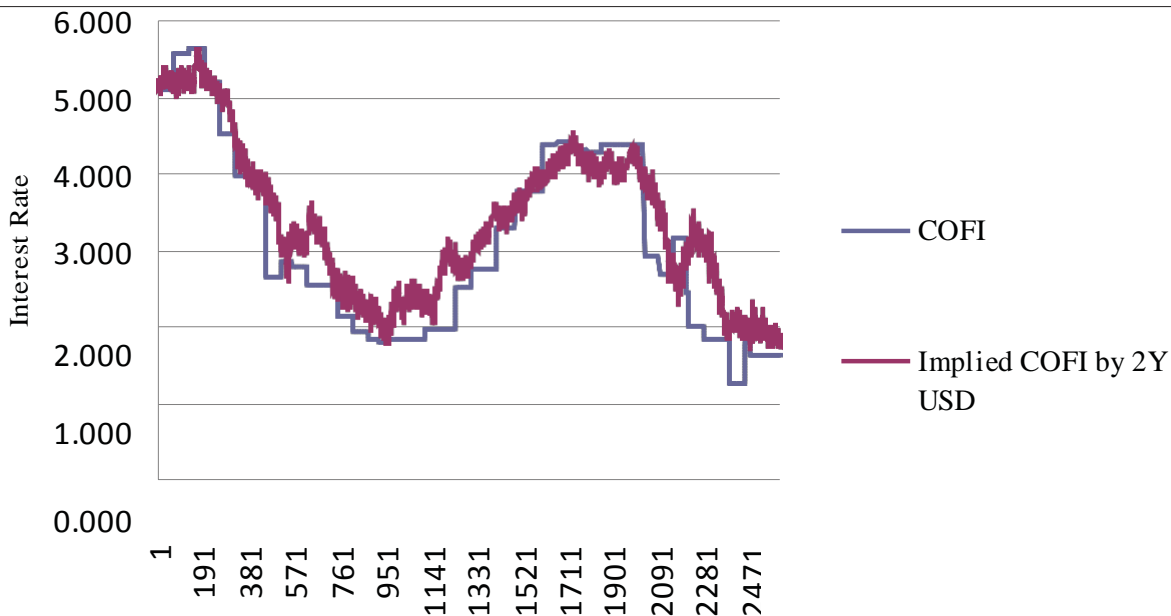
**COFI (11<sup>TH</sup> DISTRICT COST OF FUNDS INDEX)**

Using the 2 year USD swap index an appropriate fit was found using the following representation:

$$COFI = 0.575796 * USD2Y_{6month\_lag} + 1.1257$$

Where the value of USD 2Y was lagged 6 months to achieve a better fit.

COFI vs COFI Implied by USD 2Y

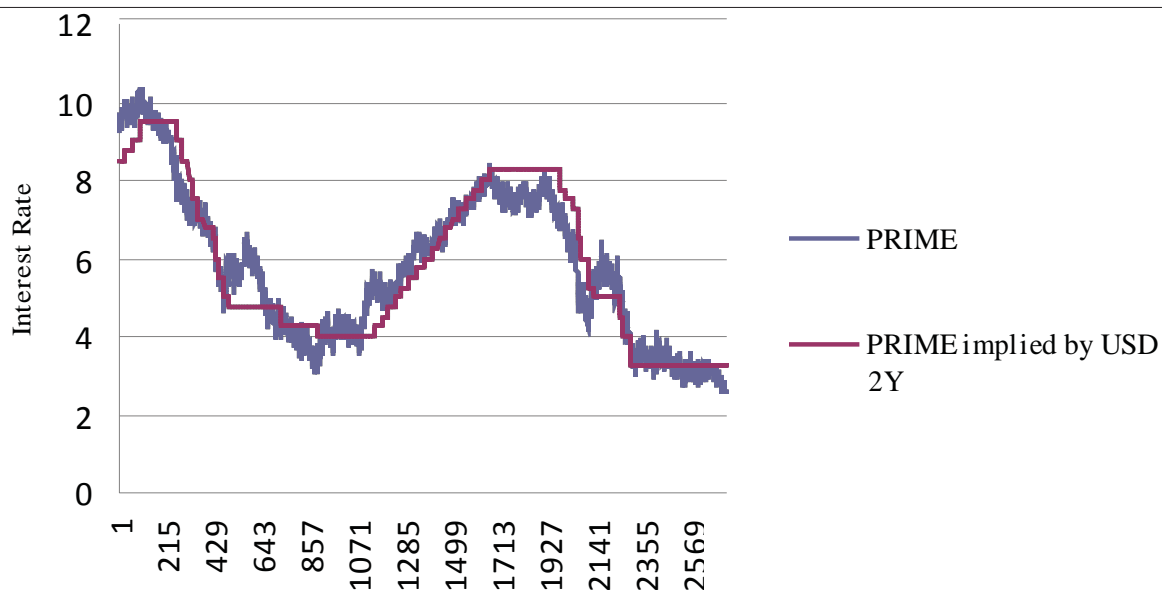


**PRIME INTEREST RATE**

The USD 2Y historical swap rate was also used to explain PRIME. Without dependence upon a lagged component, a satisfactory fit was achieved using the following formula:

$$PRIME = 1.112773 * USD2Y + 1.806723$$

Prime vs Prime Implied by USD 2Y



In each of the above cases of set rates, the curve of stresses is generated by applying the appropriate formula to the corresponding stress curve of the reference or dependent index.

### THE CASE OF EXPOSURE TO MULTIPLE INDICES IN A SINGLE DEAL

When one single deal contains exposure to multiple indices, say for example USD 1 Month LIBOR and USD 3 Year  $\Rightarrow$ , presenting a single stress curve for each index leaves in doubt the possibility that varying spreads between the indices could impact deal flow performance. Because the model produces rates in a consistent fashion, it is possible for the programming to output all path representations where each path contains all needed interest rate indices.

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